

OBTAINING THE PROPERLY WEIGHTED AVERAGE ALBEDO OF ORBITAL DEBRIS FROM OPTICAL AND RADAR DATA

D. J. Kessler¹ and K. S. Jarvis²

¹*Orbital Debris and Meteoroid Consultant, 25 Gardenwood Ln., Asheville, NC, 28803 USA*

²*Lockheed Martin Space Operations, 2400 NASA Rd 1, Mail Code C104, Houston, TX, 77058 USA*

ABSTRACT

Like any average, the proper weighting to obtain an average albedo of orbital debris depends upon the application of this average and the limitations of the data used to obtain individual orbital debris albedos. The most frequent application of the average orbital debris albedo is to transform number or flux as a function of observed orbital debris brightness into number or flux as a function of debris diameter, mass, or radar cross-section. This paper will develop and use two methods to obtain the properly weighted average albedo for transforming flux as a function of observed brightness into flux as a function of radar cross-section using data obtained from the 3.0 meter NASA Liquid Mirror Telescope (LMT) and the US Space Command sensors. The first method uses plots of number of objects detected by the LMT as a function of both magnitude and radar cross-section, corrected by assumed limitations in the detection capabilities of both the LMT and US Space Command sensors. In this method, the “proper average albedo” is the albedo required to force these two plots to predict the same number of objects at the same size. The second method integrates over the albedo distribution. Simplified assumptions allow the integration to be performed analytically. Care must be taken to consistently obtain an albedo distribution data set to either a limiting radar cross-section or a limiting magnitude, with the former being preferred. Instrument characteristics or limitations must be considered or they can contradict these simplified assumptions, resulting in misleading averages. Both techniques illustrate that the proper average albedo is not only a function of the albedo distribution, but also a function of the relative number of small debris to large debris. When the relative number of small debris to large debris is as great as has been measured for uncatalogued debris, the frequency of specular reflections is found to increase the proper average albedo significantly.

INTRODUCTION

Averages are a convenient tool to obtain “back-of-the-envelope” approximations to a problem. However, unless there is a consistently defined mathematical definition for that average, it can become a misleading tool when used to obtain more precise results. This paper will discuss the average albedo of orbital debris to be used to relate cumulative number as a function of radar cross-section (or, “radar diameter”) to cumulative number as a function of optical magnitude.

An unbiased sample of the orbital debris environment where both the radar diameter and optical magnitude were measured for each orbital debris object would permit the creation of plots of both cumulative number as a function of radar diameter and cumulative number as a function of optical magnitude. For any given number of objects, there would be both a corresponding radar diameter and an optical magnitude. Thus, by definition, the proper average albedo would be the albedo required to relate the radar diameter to the optical magnitude for any chosen number of objects. An unbiased data set does not exist, nor can it be readily obtained. In fact, if such a set existed for all sizes of orbital debris, then an albedo would not be needed to relate the two cumulative

numbers because these numbers would have been measured directly. This concept, however, forms the theoretical basis for obtaining the proper averages.

The NASA Liquid Mirror Telescope (LMT) has obtained data sets that include objects that are in the US Space Command catalogue, and also include a larger number of objects not in the catalogue, as reported by Africano et al. (1999) and Jarvis et al. (2002). When the LMT detects an object, its maximum brightness is measured and recorded. If the object is “known” and an average radar cross-section is available, this average is converted to a radar diameter using a size determination model developed by Stansbery et al. (1991) at NASA. As with many data sets, the LMT data sets contain biases. Some catalogued objects were predicted to pass through the telescope’s field of view but were not detected optically. It is not known whether these objects were too faint to be seen, or whether they failed to cross the field of view. Numerous other objects were detected optically that were not in the catalogue. It is not known whether these objects were too small to be detected by the US Space Command radars (i.e., smaller than about 10 cm), or whether the objects were detectable by the radars but not catalogued for administrative reasons. Even so, the LMT data sets represent the best data sets for obtaining an average albedo and attempting to interpret the sizes of the numerous objects detected by the LMT that were not in the catalogue. For the purpose of this study, the 1999-2000 data set was selected for use.

DATA SET OF KNOWN ORBITAL DEBRIS DETECTED BY THE LMT

Between March 6, 1999 and September 30, 2000, the LMT detected a total of 248 objects that were listed in the US Space Command catalogue. The maximum visual brightness while the object was in the telescope’s field of view was measured and then converted to “absolute magnitude”, where absolute magnitude is defined as an object’s maximum brightness in visual magnitude when observed at a range of 1000 km. The radar cross-sections of 236 of these objects were known and were converted to “radar diameter” using the NASA size determination model originally developed to analyze the Haystack radar data. Figure 1 is a scatter plot of that data, where the absolute magnitude and radar diameter of each object is plotted in terms of these derived parameters.

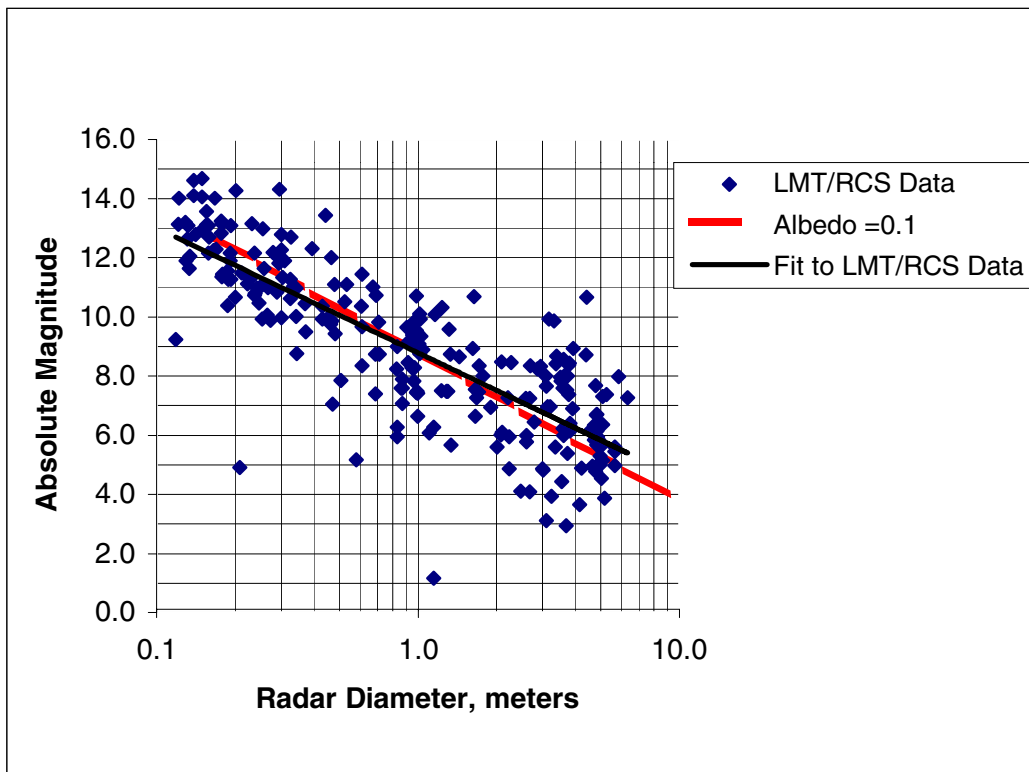


Fig. 1. Catalogued Objects Detected during the LMT 1999-2000 viewing season.

The dashed line in Figure 1 represents the theoretical relationship between absolute magnitude and radar diameter for a spherical object with an albedo of 0.1 that scatters light equally in all directions. Note that this line seems to fit the data well. The solid line is a least square fit to the data and illustrates a trend of increasing albedo with decreasing size, where objects smaller than about 1 meter will have an albedo greater than 0.1 and objects larger than about 1 meter will have an albedo less than 0.1. This apparent trend must be used with care due to the biased nature of the sample.

While both of these lines represent useful tools toward understanding the relationship between absolute magnitude and size, they do not necessarily represent a useful average. Any average used to relate size distribution to magnitude distribution must include a measure of the number of small objects relative to the number of large objects. Because a smaller object can have the same brightness as a larger object (and vice-versa), the relative abundance of sizes needs to be considered in order to use an albedo distribution to determine an appropriate average albedo. One analytical technique of obtaining this average begins by assuming the size distribution follows a particular functional form and then integrating over the albedo distribution.

ANALYTICAL TECHNIQUE FOR OBTAINING AVERAGE ALBEDO

Assume that the cumulative number, N , of orbital debris objects is given to a limiting radar diameter, D_r , by

$$N = C D_r^{-\alpha} \tag{1}$$

where C and α are constants. The value of α is the slope of a $\log N$ vs. $\log D$ plot, and may be thought of as a measure of how fast the smaller debris population is increasing with decreasing size. Assume also that the light intensity, I , of an object with an albedo of A is given by

$$I = B D_r^2 A \tag{2}$$

where B is a constant and the distribution in albedo to a limiting radar size is given by $n(A)$, which has been normalized so that $\int n(A)dA = 1$. By combining these two equations with the albedo distribution, as was done by Kessler (1972) for a meteoroid velocity distribution, the result is obtained that the number of objects to a limiting light intensity is given by

$$N = C B^{\alpha/2} I^{-\alpha/2} \int A^{\alpha/2} n(A)dA = C B^{\alpha/2} I^{-\alpha/2} A_{ave}^{\alpha/2} \tag{3}$$

where the average albedo A_{ave} is given by

$$A_{ave} = \left[\int A^{\alpha/2} n(A)dA \right]^{2/\alpha} \tag{4}$$

Therefore, the proper “average” albedo to be used in Eq. (2) when combined with Eq. (1) is an average that is weighted by the albedo to the $\alpha/2$ power. This weighting has the characteristic that as α increases so does the average albedo. The rate that it increases depends on the scatter in the albedo distribution; the larger the scatter, the more rapidly the albedo increases.

This characteristic is not the result of any physical properties of albedo. Rather, it is the result of the mathematical technique of handling a property of a size distribution in concurrence with the albedo distribution. This property has the characteristic that even though there may be a small probability that a small object will appear bright, as the number of small objects increases, they will increasingly contribute to the number of bright objects.

Since both a radar size and optical brightness for each of these 236 objects are known, a “radar size” albedo can be calculated for each object, and a plot of $n(A)$ can be made to a limiting radar diameter. Such a plot might be misleading; it would show that the largest concentration in albedo would be for an albedo less than 0.1, with a median point near this albedo. It would also show that only 10% of the objects have an albedo greater than 0.5, and another 4% have an albedo greater than 1.0, including two objects with an albedo just over 80. Since the calculated albedo assumes light is scattered equally in all directions, a calculated albedo greater than 1.0 is possible if light is preferentially scattered toward the telescope, as would occur from a specular reflection. While 4% of the albedo distribution may not appear significant, it can be very significant for large values of α , as will be shown. Therefore, rather than working with a plot of $n(A)$, the individual values of objects in the data sample were used to determine the weighted averages given by Eq. (4).

Various limiting radar diameters were selected to look at possible trends in a size-dependent albedo. Table 1 contains the resulting average albedos. The results in Table 1 indicate only a very low size-dependent albedo, and that an average albedo of 0.1 is the proper average only when the values of α are small, near 0.25 to 0.5. These values approximately correspond to the value of α obtained for the smaller catalogued objects. Objects with too small a radar size to be detected and catalogued, but with an optical brightness that is detectable by the LMT, are expected to have values of α perhaps as large as 3, corresponding to an average albedo that is more than 30 times larger than the average for smaller catalogued objects. The prediction of such large values for average albedos certainly deserves critical examination.

Diameter (num) / $\alpha=$	0.25	0.5	1.0	2.0	3.0	4.0
1.0 m (114)	0.08	0.10	0.17	0.92	3.63	7.9
0.5 m (152)	0.10	0.12	0.19	0.82	3.08	6.9
0.2 m (201)	0.11	0.14	0.23	1.08	3.94	8.3
0.12 m (236)	0.12	0.14	0.22	0.96	3.56	7.7

A look at Figure 1 suggests a cause for these high average albedos. There are a number of data points where the absolute magnitude is several visual magnitudes away from the “albedo = 0.1” line. Those above the line have albedos less than 0.1 and those below the line have albedos greater than 0.1. The limiting magnitude of the LMT also limits the number of data points above the line and could contribute to generating albedos that appear to increase with decreasing size; however, this limit does not appear to significantly affect the average albedo. The same cannot be said about those data points below the line.

Two of the objects (data points 1.1, 1.2 and 0.21, 4.9) have average albedos just above 80. This suggests that these two observations were more characteristic of a specular reflection rather than a diffuse reflection. Nine other objects have albedos between 1.0 and 10, and may also have specular reflection contributions. However, removing only the two extreme data points from the sample very significantly changes the average albedos for larger values of α . For example, removing the two extreme data points only changes the average albedo for the smallest debris from 0.22 (in Table 1) to 0.15 when $\alpha = 1$; the change is even less for smaller values of α . However, when $\alpha = 2$ the average albedo is decreased from 0.96 to 0.26; when $\alpha = 3$ the average is decreased from 3.56 to 0.44. Therefore, these two data points, representing less than 1% of the sample, are capable of changing the deduced radar diameter of a size distribution where $\alpha = 3$ by a factor of 2.8 ($=\sqrt{3.56/0.44}$).

The implications of this are significant toward interpreting a measured magnitude distribution in terms of debris sizes, if this albedo distribution is typical of smaller debris. Only one of the eleven objects with albedos greater than one was a debris fragment; the remaining objects were high-altitude payloads; consequently this albedo distribution may not be representative of smaller debris. However, the distribution illustrates how a larger number of small debris objects, objects normally much too small to be detected optically, might be capable of producing a sufficient number of detectable specular reflections and confused with larger objects. This phenomenon is also illustrated when using a graphic technique to determine average albedo.

GRAPHIC TECHNIQUE FOR OBTAINING AVERAGE ALBEDO

From the above theoretical predictions, if an albedo of 0.1 is assumed and used to predict the “optical diameter” for the optically detected catalogued objects, then a plot of resulting number vs. “optical diameter” and a plot of number vs. radar diameter should compare favorably, at least for the smaller catalogued objects. Figure 2 is such a plot. Notice in Figure 2 that the two curves agree very well for sizes smaller than about 2 meters, where the value of α is about 0.4. However, for larger sizes, where the value of α begins to increase, the fit is not as good and suggests a larger albedo is necessary to obtain agreement. For example, at the sizes where the cumulative number is 3, the optical size needs to be reduced by a factor of 2.4 to be in agreement with the radar size. This requires an increase in the assumed albedo by a factor of 5.8, or an average albedo for this part of the

curve of 0.58. The curves predict the necessity for an increasing average albedo with increasing size in the region where α is large.

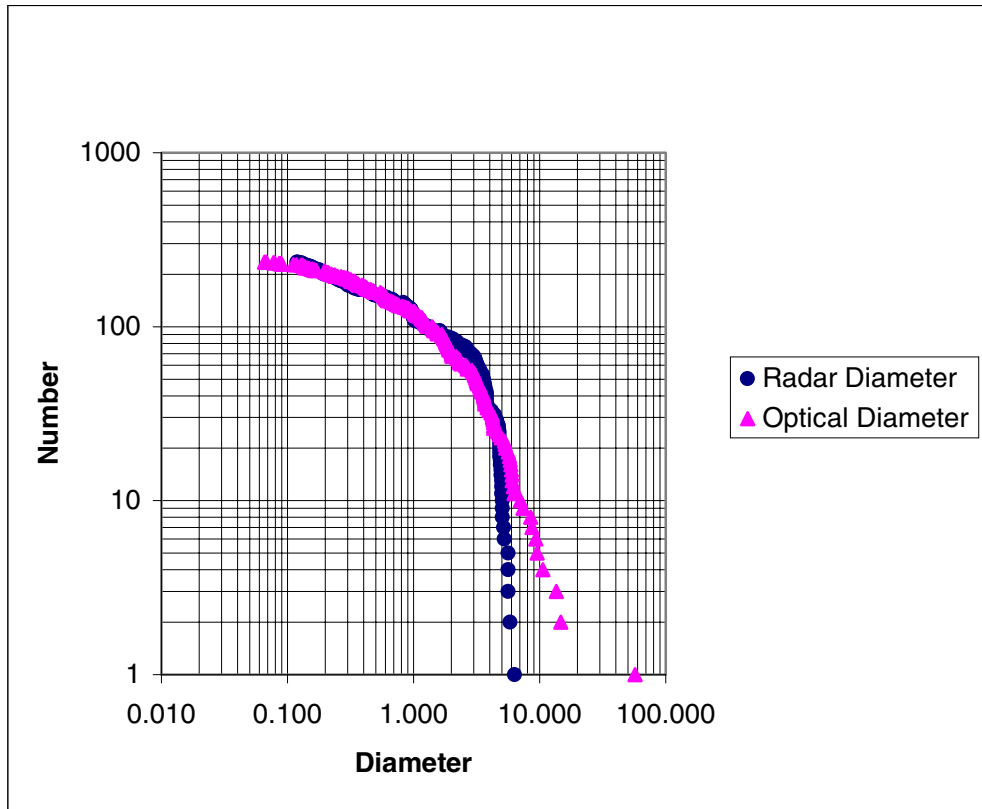


Fig. 2. Comparison of radar diameter and optical diameter for catalogued objects, assuming a 0.1 albedo.

This increasing average albedo with size is, of course, actually a way of handling the specular reflection phenomenon discussed in the previous section. The value of α is so large for the 30 largest objects that assuming all 30 objects have exactly the same size of about 5 meters could accurately approximate this part of the radar size distribution. Under this model, Figure 2 can be interpreted as concluding that an average albedo of 0.1 works very well for all 30; however, because of the albedo distribution (which is partly controlled by specular reflections), 3 (10%) of them appear to be at least 2.4 times larger and 1 (3%) appears to be 8.9 times larger.

The different shape in the optical and radar size distribution for these larger objects also illustrates the different ways that radar diameter and optical diameters were obtained. Radar, like optical, returns signals from irregularly shaped objects that can vary greatly, depending on the orientation and shape of the object. However, the radar measurements used here have both been averaged over long periods of time and run through a size determination model which corrects for the effective values of α , while the optical measurements only represent the maximum brightness recorded during a very short observing period. Consequently, the resulting radar sizes given here are expected to be much closer to the actual size of the objects detected, while the optical sizes are more scattered. Given the importance of this scatter in optical sizes, especially as optical techniques are used to sample even smaller objects, a quick theoretical look at the implications of specular reflections is appropriate.

Implications of Specular Reflections to Size Distributions

In order to consider some of the implications of specular reflections, consider the following hypothetical case: Assume that you have a known distribution of metal spheres in space with sizes 10 cm and larger. We plan to use optical sampling techniques to measure the distribution of smaller objects, but do not know whether the smaller objects are spheres or flat metal plates. If they are flat metal plates, each will reflect light as a mirror would, with all of the light concentrated in a cone, $\frac{1}{2}$ degrees in diameter. This means that a flat metal plate that has a cross-

sectional area that is 5×10^{-6} that of a metal sphere will have the same brightness as that metal sphere, if the observer happens to be in the $\frac{1}{2}$ degree cone of light of the flat plate. The probability that the observer is located in that cone during any instant in time is also 5×10^{-6} . Therefore, an object that is only 0.022 cm in diameter might have the same brightness as a 10 cm sphere; however, because of the low probability of seeing any particular 0.022 cm flat plate, in order to detect the same number of 0.022 cm flat plates as 10 cm spheres, there would have to be 2×10^5 times as many 0.022 cm objects as 10 cm objects. If these numbers of objects were part of a distribution expressed by Eq. (1), then α would have a value of 2.

We could also assume that the flat plates were slightly curved, so that the light was scattered in a larger cone-angle. This would increase the size detected to something larger than 0.022 cm, but lower the number of these objects required to equal the 10 cm sphere detection rate. However, no matter what cone-angle the light is scattered, the value of α would remain unchanged. Therefore, if α has a value of 2 or greater, as has been determined from measurements of debris smaller than 10 cm, then the possibility of specular reflections significantly biasing those measurements must be considered. This bias is handled in the average albedos given in Table 1, where the average albedo increased by more than a factor of 4 as α increased from 1 to 2. However, this assumes that the frequency of specular reflections in this LMT data sample is representative. As was shown earlier, only two objects in the data sample significantly influence the results in Table 1 when α is 2 or greater. Therefore, a larger sample is likely to be required to adequately describe the consequences of specular reflections.

APPLICATION OF AVERAGE ALBEDO TECHNIQUES TO DEBRIS OBJECTS

These techniques to determine average albedos will now be applied to orbital debris detected by the LMT in low Earth orbit. Three data sets will be used: 1. Catalogued objects detected, or “correlated targets” (CTs). 2. Catalogued objects predicted to be detected optically, but not seen (nosees). 3. Objects detected by the LMT that cannot be correlated with any catalogue object, or “uncorrelated targets” (UCTs). To minimize observational bias in the data, all objects above 2000 km were eliminated from the samples. Although meteors were removed from the UCT data set using criteria that included angular velocity and Earth shadow-height, this sample still contained some meteors that could not be distinguished from low altitude orbiting objects. In order to eliminate nearly all of these remaining meteors, all objects below 600 km were removed from all data sets. Finally, all objects identified in the catalogue as being either a payload or rocket body were eliminated. The resulting data set included 81 CTs (no RCS was available for 7 of these), 593 UCTs, and 43 nosees (no RCS was available for 9 of these).

Scatter Plot of Debris Objects

Figure 3 is a scatter plot of the 74 CTs where an RCS was available. Notice that the two data points in Figure 1 that had albedos just above 80 are not in this data set, so that specular reflections will not influence average albedos as greatly as in Table 1; however, a larger sample might contain larger albedos.

Average Albedo of Debris Objects Using Analytical Technique

With only 74 objects to obtain an average albedo, the sample cannot be accurately divided into size intervals as was done in Table 1. Therefore, the averages are only determined for the entire sample, and are shown in Table 2. Notice that for values of α of 1.0 or less, the average albedos are not too different than those shown in Table 1. However, the averages in Table 2 are much less than in Table 1 for $\alpha \geq 2$. This is almost totally due to the absence of the two high albedo data points previously mentioned. Their absence could be explained by either the definition of the sample (i.e., only payloads produce such an extreme albedo), or the statistics of a smaller sample (i.e., less than 1% of the objects are expected to have such an extreme albedo). The correct explanation must await the results of future measurements and analysis.

Average Albedo of Debris Objects Using Graphic Technique

By removing payloads and rocket bodies, a plot of cumulative number vs. diameter is more consistent with the functional form of Eq. (1). In addition, the value of α in such a plot is larger than the value for the smaller debris in Figure 2, having a value of about 1.5 for the CTs. From Table 2, the average albedo should be between 0.15 and 0.22; using the data, a calculation for $\alpha=1.5$ obtained an average albedo of 0.18. Figure 4 assumes an albedo of 0.18 to calculate the optical diameter.

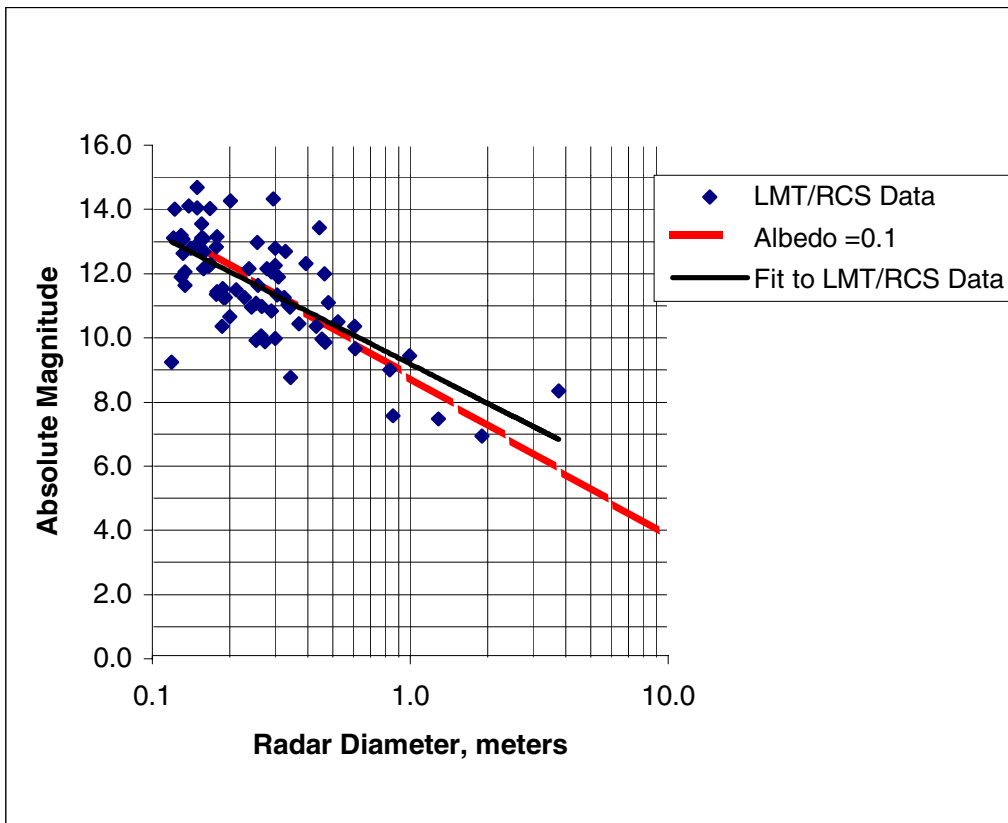


Fig. 3. Catalogued debris objects detected by the LMT between 600 km and 2000 km during the 1999 - 2000 viewing season.

Table 2. Average albedo for various values of α , based on a sample of 74-catalogued objects between 600 km and 2000 km with payloads and rocket bodies removed						
$\alpha =$	0.25	0.5	1.0	2.0	3.0	4.0
Average albedo =	0.12	0.13	0.15	0.22	0.36	0.58

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The agreement between the two curves is fairly good at the smaller debris sizes where the value of α is about 1.5. However, at the smallest sizes, the optical data is constrained to include only catalogued debris, so it becomes incomplete and departs from the radar data. A more accurate comparison at this point would be to include all of the debris detected by the LMT (i.e., the CTs, including those with no RCS, plus the UCTs) in the optical data set. In order to make the radar data totally consistent with the optical data, the nosees were added to the radar data since they were predicted to pass through the LMT field of view, and could have done so, but were not detected optically. Finally, a radar diameter of 0.15 m was arbitrarily assigned to those 16 catalogued debris objects that did not have an RCS. The results are shown in Figure 5; again with an assumed albedo of 0.18.

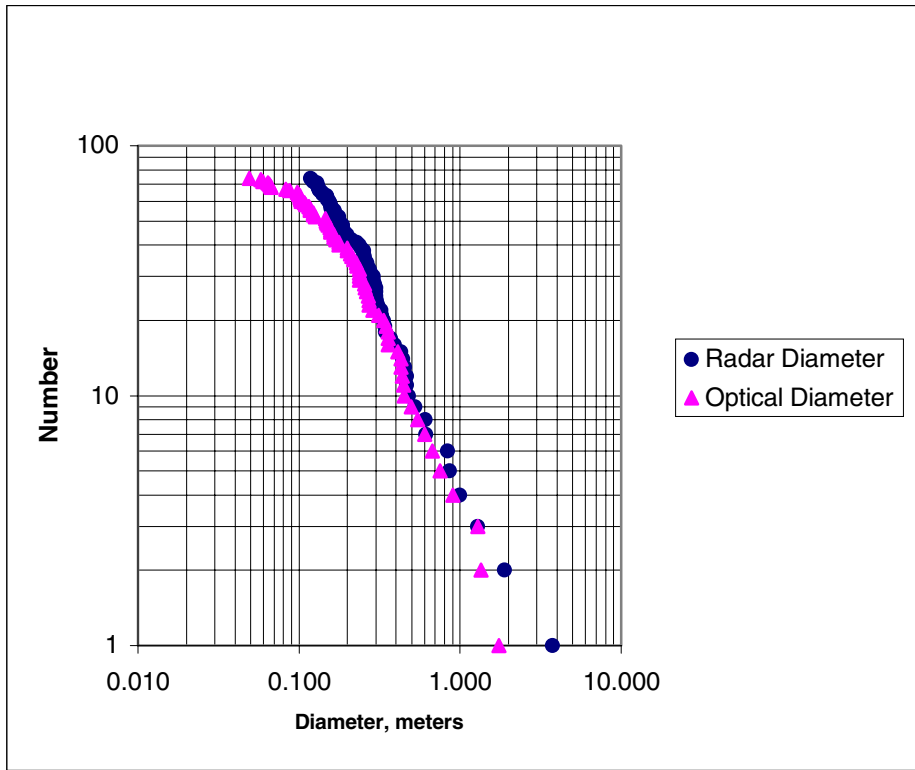


Fig. 4. Comparison of radar diameter and optical diameter, for catalogued debris between 600 and 2000 km, assuming a 0.18 albedo.

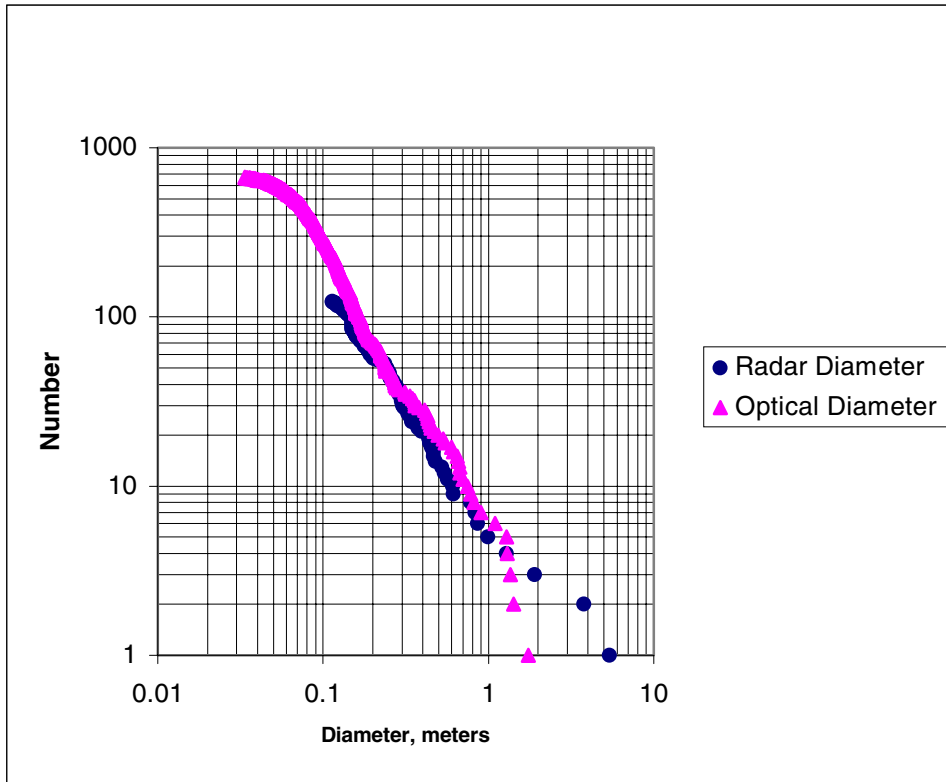


Fig. 5. Comparison of radar diameter and optical diameter, for all debris detected, or expected to be detected, by the LMT between 600 and 2000 km, assuming a 0.18 albedo.

Except for a few larger fragments, the agreement between the optical and radar data in Figure 5 is fairly good. A slightly larger albedo of about 0.25 might be appropriate for sizes between 0.2 m and 1 m, and a much smaller albedo would be required for the few large fragments; however, 0.18 works fairly well for the whole distribution. The optical curve shows that the value of α is increasing with decreasing size, obtaining a value of 2.1 before beginning to decrease, almost certainly because of the limiting magnitude of the LMT. Other orbital debris data suggests that the value of α will approach 3.0 for smaller debris objects. Therefore, the proper average albedo for objects smaller than 20 cm is greater than 0.18. The results in Table 2 conclude that the average albedo for these smaller objects should be between 0.23 and 0.36.

Without changing the albedo for sizes smaller than 20 cm one would erroneously predict from Figure 5 that the number of debris objects 10 cm and larger detected by the LMT is 2.3 times the number of debris objects in the catalogue. However, the proper albedo for this size is at least 0.23, which reduces the LMT sizes and changes the 10 cm number to 1.9 times the debris catalogue. A higher albedo in this region could be justified either from a greater value of α in Table 2, or the use of Table 1. For example, the possible albedo of 0.36 reduces the LMT debris number to only 1.2 times the debris catalogue. Therefore, questions concerning the exact completeness of the catalogue to 10 cm must await additional data.

The smaller value of α at slightly larger sizes produces less uncertainty in the relationship between the optical and radar data near 20 cm. The LMT data suggests more confidently that the catalogue could be fairly complete somewhere between 15 cm and 20 cm, depending on the actual sizes of the 16 objects where no RCS was available.

CONCLUSIONS

When transforming number as a function of observed orbital debris brightness into number as a function of orbital debris diameter, the proper average albedo not only depends on the physical characteristics of the debris, but on the relative number of small debris to large debris. This relative number can be expressed by the value of α when the cumulative number varies with diameter to the negative α power. As α increases, so does the proper average albedo. When $\alpha \leq 1$, the proper average albedo is between 0.1 and 0.2 and not very sensitive to the frequency of specular reflections. When $\alpha \geq 2$, the proper average albedo is a sensitive function of both the value of α and the frequency of specular reflections. If this frequency is as great as was measured by the LMT for payloads, then the proper average albedo when $\alpha \geq 2$ is greater than 1, and increases rapidly with increasing α . Therefore specular reflections may be very important to understanding the debris sizes when $\alpha \geq 2$.

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Email address of D.J. Kessler: Dkessler@charter.net

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