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COLLISION PROBABILITY AT LOW ALTITUDES RESULTING FROM ELLIPTICAL ORBITS

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ABSTRACT

The collision probability between a spacecraft and another object in Earth orbit can be expressed as a function of the orbital perigee, apogee, and inclination of the object. Usually, the probability is not a sensitive function of inclination. Collision can only occur when the spacecraft is located at an altitude which is between the perigee and apogee altitudes of the object. The probability per unit time is largest when the perigee and apogee are nearly equal (i.e., the orbit is nearly circular). Therefore, it is usually concluded that objects in circular orbits represent the greatest hazard to other spacecraft. However, at low altitudes, atmospheric drag causes perigee and apogee to change with time, such that circular orbits have a much shorter lifetime than many of the elliptical orbits which pass through the lower altitudes. Consequently, when the collision probability is integrated over the lifetime of the orbiting object, some elliptical orbits are found to have a much higher total collision probability than circular orbits. Objects in these elliptical orbits could represent the greater source of hazardous objects to spacecraft operating in low Earth orbit. Some common objects in these elliptical orbits are rocket bodies used to boost payloads from low Earth orbit to geosynchronous orbit.

INTRODUCTION

A common assumption when calculating collision probabilities between orbiting objects is that the objects are in circular orbits. This assumption is common because the calculation is easier, and in Earth orbit, most orbiting objects are in near circular orbits. Since circular orbits spend all of their time at a particular altitude, their contribution to the collision probability at that altitude is much higher than an elliptical orbit passing through the same altitude.

However, at low altitudes in Earth orbit, orbital lifetimes are relatively short; therefore, to maintain a constant collision probability, or flux, a constant source of objects is required. This source can be from new launches into these altitudes, or it can be as the result of older objects dragging down from higher altitudes. Because this constant source is required, the orbital lifetime of the objects resulting from the source is also important. Since elliptical orbits which pass through a particular altitude can have a much longer orbital lifetime than circular orbits at that altitude, these elliptical orbits may require a smaller source of objects to maintain a particular flux.

This paper will discuss the equations used to calculate the collision probabilities resulting from elliptical orbits, and the required production rate of objects to explain the flux measured from analysis of the returned Solar-Max surfaces and the STS-7 window pit. These production rates are shown to be small and reasonable, especially if the source is in certain elliptical orbits.

CALCULATING COLLISION PROBABILITIES

Collision probabilities can be calculated by first calculating the spatial density resulting from an object, then using that spatial density to calculate flux and collision probability /1/. Spatial density, S, is the probability of finding an object with orbital elements of inclination i, perigee q, and apogee q′ within a volume element located R distance from the center of the Earth, and at latitude B, and is given by

$$S(R,B) = s(R)f(B) \tag{1}$$

where s(R) is the spatial density averaged over all latitudes, and given by

$$s(R) = \frac{1}{2\pi^2 R(q+q')[(R-q)(q'-R)]^{1/2}} \tag{2}$$

and f(B) is the ratio of the spatial density at B to the spatial density averaged over all latitudes, and given by

$$f(B) = \frac{2}{\pi(\sin^2 i - \sin^2 B)^{1/2}} \tag{3}$$

The orbital inclination of objects in low Earth orbit varies from near zero to 145 degrees. This distribution of inclination produces a spatial density distribution with latitude which is nearly constant (within a factor of 2). Consequently, for most calculations the approximation that f(B)=1 is appropriate, and will be used here.

Flux, F, or the number of impacts per unit cross-sectional area per unit time, is given by

$$F = Sv \tag{4}$$

where v is the relative collision velocity. The average number of impacts, N, (or the collision probability, if N is small) is given by

$$N = \int FAdt \tag{5}$$

where A is the collision cross-sectional area exposed to the flux for an increment of time dt.

SOURCE STRENGTHS REQUIRED FROM CIRCULAR ORBITS

Assume that G particles per unit time are produced from an object in circular orbit, at some distance above R. Assume also that as the orbits of these particles decay from atmospheric drag and pass through R, remaining in the increment of altitude dR for an increment of time dt, and an equilibrium spatial density, S, is established at R. Then from the definition of G and S,

$$G = 4\pi R^2 S dR/dt = 4\pi R^2 F v_d / v \tag{6}$$

where v_d = dR/dt, or the rate of change of the orbital semi-major axis due to atmospheric drag, and F is the measured flux at R from particles having relative velocity v.

As an application of this equation, analysis of Solar-Max louver surfaces gives an orbital debris flux of 1×10^{-6} gm (about 0.1 mm diameter) debris of about $0.3/\mathrm{m}^2$-yr. These impacts were predominantly from paint particles /2/. The average cross-section of a randomly tumbling surface is 1/4 its surface area, so that F in equation 6 is $1.2/\mathrm{m}^2$-yr. At 500 km altitude (R = 6878 km), and during average solar activity, a 1×10^{-6} gm particle will decay at a rate of about 26 km/day. Assuming an average collision velocity of 9 km/sec. gives a required production rate of 2.4×10^{10} particles per year, or about 24 kgm/year of 0.1 mm particles. Other size impacts were also measured. The size distribution of these impacts would suggest that a slightly larger production rate is required at smaller sizes. Over the entire size distribution measured by Solar-Max, a particle production rate, assuming circular orbits, of about 100 kgm/yr is required. The effects of atomic oxygen on several hundred painted spacecraft just above Solar-Max altitude would give this this production rate.

SOURCE STRENGTH REQUIRED FROM ELLIPTICAL ORBITS

The average number of collisions from an object in an elliptical orbit can be found by combining equations 4 and 5. Since collision velocity is mostly a function of inclination, v can be taken out of the integral, along with the collision area A. Consequently, for elliptical orbits,

$$N_e = Av\int Sdt \tag{7}$$

The value of S is a function of time since both perigee and apogee will vary due to atmospheric decay; when the apogee is above about 5000 km altitude, lunar and solar perturbations will also cause the perigee and apogee to vary. Numerical techniques must be used to determine how these orbital parameters change with time. The same program used in /3/, which was developed by Alan Mueller, will also be used here. This program combines gravitational perturbations with equations developed by King-Hele /4/.

The average number of collisions from an object in a circular orbit can be expressed the same way, except that a singularity occurs when R = q = q'. This singularity is integrable, so that the collision frequency for a circular orbit is given by

$$N_c = Av/(4\pi R^2 v_d) \tag{8}$$

The ratio N_e/N_c is then the ratio of the source required from elliptical orbits to the source required from circular orbits to maintain a given flux. This ratio, W, is then given by

$$W = \int Sdt(4\pi R^2 v_d) \tag{9}$$

The value of W was determined by numerical integrating equation 9 using the decay model in /3/. Figure 1 shows how the perigee and apogee varied as a function of time for the case where the particle diameter was 1 cm, initial perigee was 400 km, and initial apogee was 4000 km. A circular orbit is also shown. Note that the amount of time that the elliptical orbit passes through 400 km is about 6000 days, or 16 years, compared to less than 10 days for a circular orbit to drop more than 10 km, and have no possibility of collision with an object at 400 km.

The calculated value of W is given in figures 2 through 7. Figures 2 and 3 represent cases where the initial perigee height is equal to the altitude of interest; apogee heights were kept below 5000 km, which means that gravitational perturbations, and hence inclination, are not important. These types of conditions give the largest values for W. As seen in figures 2 and 3, apogees above 2000 km are more than 10 times more effective in producing a given flux than circular orbits. The Solar-Max measurements would require a particle production rate of less than 10 kgm/yr if the source were in this type of elliptical orbit. However, spacecraft in elliptical orbits with perigees near 500 km and apogees larger than 2000 km are not common, although explosion fragments can be found in these types of orbits.

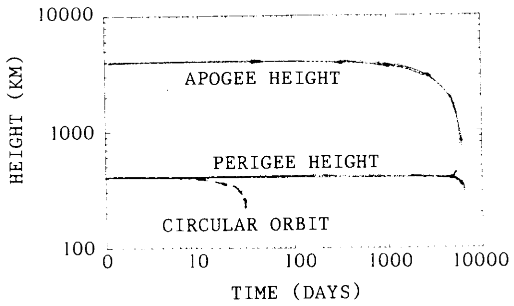

Fig. 1. Decay profile for 1 cm dia., 2 gm/cm^2 particle: Elliptical and circular orbit.

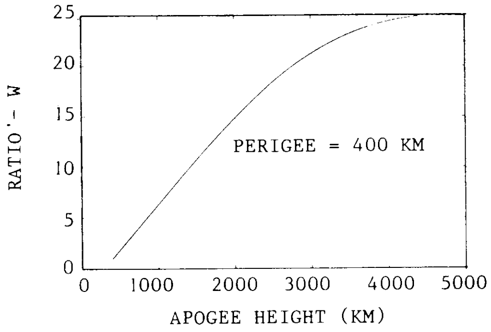

Fig. 2. Required production rate to maintain a given flux at 400 km altitude ratioed to a source in circular orbit.

A more common elliptical orbit results from placing payloads into geosynchronous orbit. When the Shuttle is used, an upper stage (IUS or PAM) is left in an orbit with a perigee near 300 km altitude, an apogee near 36000 km, and inclination of 28.5 degrees. The Ariane leaves its 3d stage in an orbit with a perigee near 200 km, apogee near 36000, and inclination of about 7 degrees. Depending on the initial sun angle, these types of orbits can represent a potentially significant source of particles for spacecraft operating below 500 km altitude as seen in figures 4 through 7.

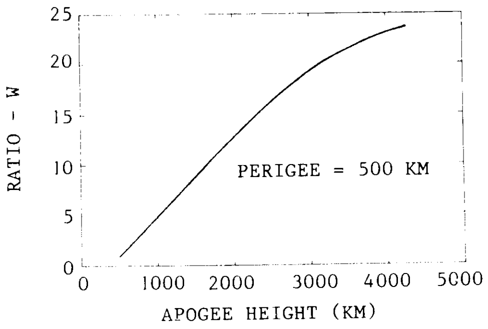

Fig. 3. Required production rate to maintain a given flux at 500 km altitude ratioed to a source in circular orbit.

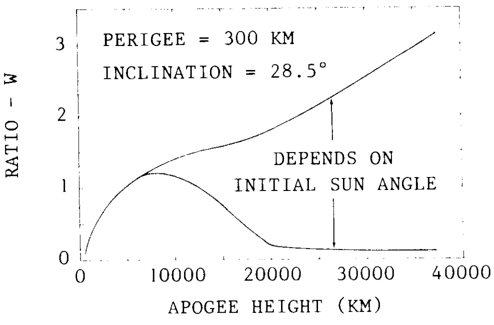

Fig. 4. Required production rate to maintain a given flux at 400 km altitude ratioed to a source in circular orbit.

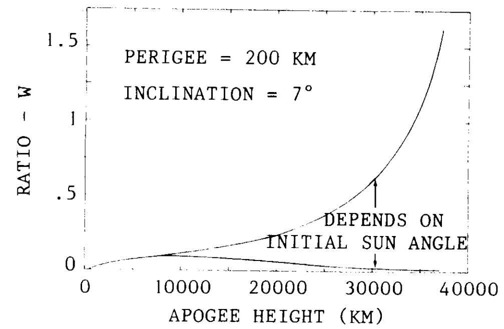

Fig. 5. Required production rate to maintain a given flux at 400 km altitude ratioed to a source in circular orbit.

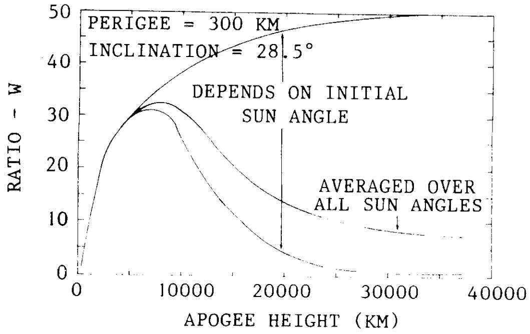

Fig. 6. Required production rate to maintain a given flux at 300 km altitude ratioed to a source in circular orbit.

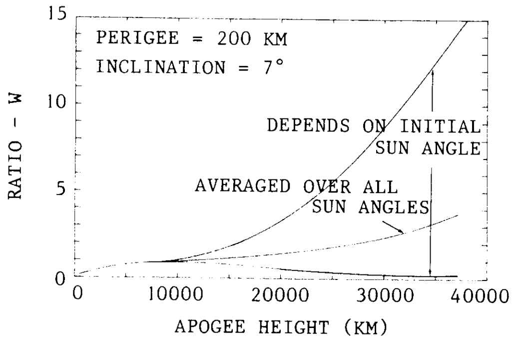

Fig. 7. Required production rate to maintain a given flux at 300 km altitude ratioed to a source in circular orbit.

The initial sun angle (an angle which is a function of the position of the sun and orientation of the orbital plane at the time of launch) determines whether solar perturbations will increase or decrease the orbital lifetime of elliptical orbits with apogees greater than 5000 km altitude. As seen in figure 6, at 300 km, some sun angles can be more than 50 times more efficient in maintaining a given flux than circular orbits. Since the value of v_d is about 50 times larger at 300 km than at 500 km, the same flux measured on Solar-Max at 500 km could be maintained at 300 km from a source of about 100 kgm/yr, if the sources were in elliptical orbits which gives these high values for W. This could explain the STS-7 window pit /5/. That is, the flux represented by this impact on the STS-7 window at 300 km would require a very large source of 0.2 mm paint particles, if the impact originated from an object in circular orbit. However, a much more reasonable source is required if the impact originated from an object in an elliptical orbit having a high value of W.

CONCLUSIONS

At low altitudes, objects in elliptical orbits can represent a more important source of orbital debris than objects in circular orbits. The long orbital lifetime of upper stages currently being left in orbit after placing payloads into geosynchronous orbit could represent the dominant source of debris at altitudes below 500 km.

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