Average Relative Velocity of Sporadic Meteoroids in Interplanetary Space

Donald J. Kessler*
NASA Manned Spacecraft Center, Houston, Texas

Introduction

THE orbital elements obtained from more than 2000 sporadic meteors listed by McCrosky and Posen[1] can, with certain assumptions, be used to determine the meteoroid velocity distribution relative to a body in orbit around the sun. The purpose of this Note is to determine the average velocity of meteoroids relative to a spacecraft having any specified velocity and position in space.

Correction of Data to a Limiting Mass

Before any velocity calculations are made, the meteor sample must be corrected to some limiting mass. The major selection effects can easily be considered by the following procedure.

Assuming that the number of meteors in Ref. 1 can be expressed by

$$N \propto I_f^{-\beta} n_f(v_\infty) \tag{1}$$

where I_f is the maximum intensity of the meteor as recorded on the film, and $n_f(v_\infty)$ is the number of meteors with velocities between v_∞ and $v_\infty + dv_\infty$, and v_∞ is the accelerated velocity of the meteor at the instant the meteor enters the atmosphere of the earth.

The maximum intensity of a meteor is given by Ref. 2 as

$$I_{max} \propto m^{0.9} v_\infty^{3.5} \tag{2}$$

where m is the initial meteor mass. Then, the intensity of the meteor as recorded on the film will be

$$I_f \propto I_{max}/v_a h^2 \tag{3}$$

where v_a is the angular velocity of the meteor and h is the altitude of the meteor above the observation station. Assuming that the radiant distribution of the meteors is independent of the velocity and intensity distributions

$$v_a \propto v_\infty/h \tag{4}$$

Reference 3 relates v_∞ to h by

$$h \propto v_\infty^{0.25} \tag{5}$$

The number of meteors per unit area will vary as

$$F \propto N/h^2 \tag{6}$$

Thus, combining Eqs. (1) to (6), the flux as a function of mass and velocity will vary as

$$F \propto m^{-0.9\beta} n_f(v_\infty)/v^{2.5\beta+0.25(2-\beta)} \tag{7}$$

Therefore

$$n_m(v_\infty) = n_f(v_\infty)/v^{2.5\beta+0.25(2-\beta)} \tag{8}$$

where $n_m(v_\infty)$ is the velocity distribution to some limiting mass.

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* Aerospace Technologist, Meteoroid Sciences Branch.

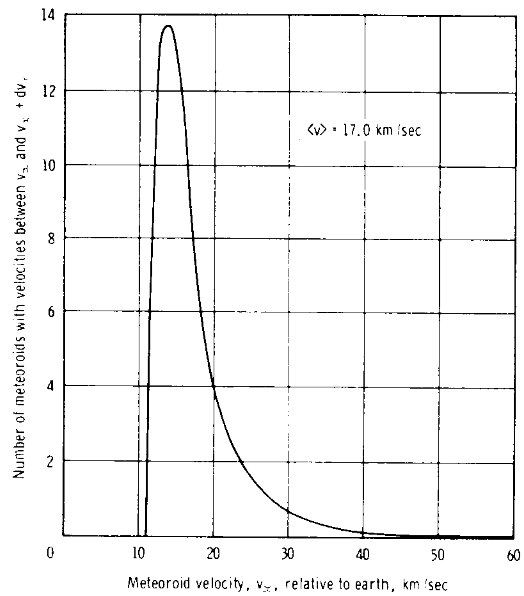

Fig. 1 Meteoroid velocity distribution relative to earth, after gravitational attraction.

Reference 4 gives the value of β as 1.34, so that the weighting factor becomes $v_\infty^{-3.5}$. Although the method of obtaining the weighting factor of $v_\infty^{-3.5}$ is simplified, this factor can be shown to be equivalent to using the more precise method of velocity-distribution correction used by Dohnanyi.[5] The results differ slightly because Dohnanyi found $N \propto m^{-1}$ (i.e., $\beta = 1/0.9$) and did not consider v_∞ as a function of h.

The data for the sporadic meteors in Ref. 1 were put on magnetic tape and a computer program was written to obtain $n_m(v_\infty)$. The results are shown in Fig. 1, with an average velocity of 17.0 km/sec.

Velocity Distribution Relative to a Massless Earth

Since the earth "pulls in" more slowly moving meteoroids, each meteor must be weighted by v_G^2/v_∞^2 (as given in Ref. 6)

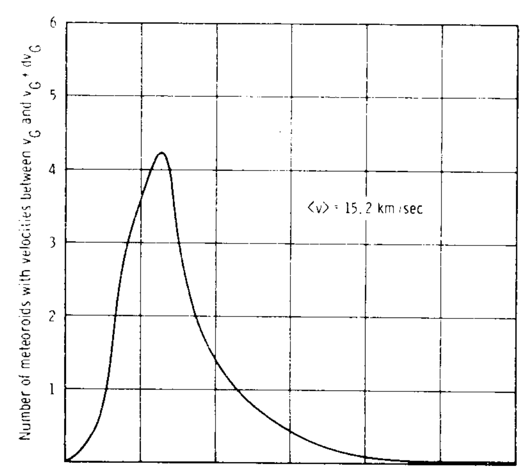

Fig. 2 Meteoroid velocity distribution relative to earth, before gravitational attraction.

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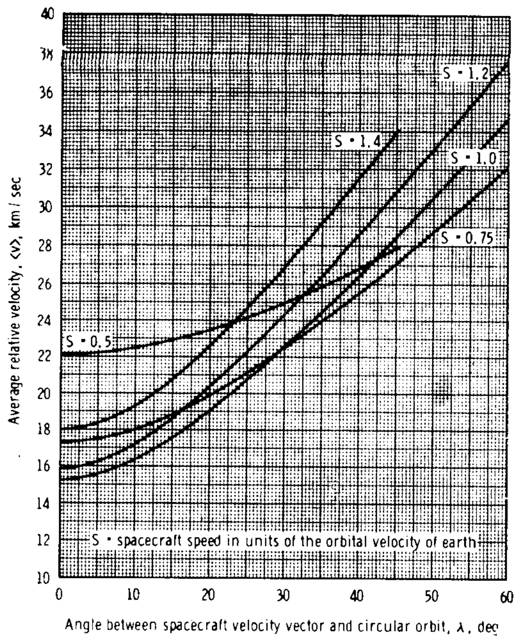

Fig. 3 Average meteoroid velocity relative to a spacecraft at 1.0 a.u.

in order to determine the velocity distribution relative to a massless earth; v_G is the meteoroid velocity relative to a massless earth. The resulting geocentric velocity distribution is shown in Fig. 2, with an average velocity of 15.2 km/sec.

An incidental result of producing Figs. 1 and 2 is that the ratio of the sum of all weighting factors used to produce Fig. 1 to the sum of the weighting factors used in Fig. 2 is equal to the ratio of the meteor flux on the surface of the earth to the meteoroid flux outside the gravitational sphere of influence of the earth. This ratio is equal to 2.1.

Average Velocity Relative to a Noncircular Orbit

Assume that a spacecraft is at 1 a.u. from the sun and outside the gravitational sphere of influence of the earth and that the spacecraft has a heliocentric speed of S (in units of the orbital velocity of the earth). The velocity vector of the spacecraft is in the ecliptic plane, making an angle λ with the orbital path of the earth. Then from the semimajor axis a, inclination i, and eccentricity e of each meteor as given in Ref. 1, the intersection velocity v can be computed by using standard orbital mechanics. However, the weighting factor v/v_G must be added to consider the differences in the probability of collision for circular and noncircular orbits.[7] Thus, the total weighting factor becomes $vv_G/v_\infty^{5.5}$.

Velocity distributions were obtained for various values of S and λ, and the average velocity $\langle v \rangle$ was then obtained for each distribution. Figure 3 gives $\langle v \rangle$ as a function of λ for various values of S. At $S = 1.0$ and $\lambda = 0.0$, $\langle v \rangle$ is the same on Fig. 3 as on Fig. 2.

Figure 3 can be used to determine $\langle v \rangle$ for any distance from the sun by making the assumption that the distribution of a/R, e, and i for meteoroids is independent of the distance from the sun R. In this general case, S is redefined to be the ratio of the spacecraft speed at R to the circular orbit speed at R, and λ is the angle between the spacecraft velocity vector and a circular orbit at R. The average relative meteoroid velocity will then vary as $R^{-0.5}$ for constant values of S and λ. For example, if a spacecraft is in orbit around the sun at $R = 1.25$ a.u. and if the orbit has a perihelion of 1.0 a.u. and an aphelion of 1.5 a.u., then $S = 1.0$, $\lambda = 11.5°$, and (from Fig. 3) $\langle v \rangle = 16.6/(1.25)^{1/2} = 14.8$ km/sec.

It is unlikely that a spacecraft would have values of S greater than 1.4 or less than 0.5. A value of $S = (2)^{1/2}$ corresponds to a parabolic orbit, and $S = 0.5$ is the speed of a spacecraft at the aphelion of a highly elliptical transfer orbit having an eccentricity of 0.75. A value of $S = 1.0$ corresponds to the spacecraft speed when the spacecraft is at a distance from the sun equal to the semimajor axis of the transfer orbit. Thus, Fig. 3 can be used to determine the average relative meteoroid velocity for most interplanetary spacecraft.

References

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