Long Division and Synthetic Division

By: Douglas A. Ruby
Class: Algebra II – H Period

INSTRUCTIONAL OBJECTIVES:

At the end of this 2- part lesson, the student will be able to:

1. Use long division to divide one polynomial by another polynomial of lesser degree (typically a binomial).
2. Use synthetic division to divide a polynomial by an arbitrary binomial \((x-c)\) having a leading coefficient of one \((1)\). Also use synthetic division where the binomial has an arbitrary coefficient in the form \((ax - c)\).
3. Interpret answers to long division (or synthetic division) of a polynomial by the binomial \((x-c)\) as: 
   \[ax^n + bx^{n-1} + \ldots + \frac{r}{x-c}\]
   where \(r\) is the remainder.
4. Correctly interpret answers to the expression \((ax^n + bx^{n-1} + \ldots) \div (x-c)\) with a remainder \(r = 0\) as having factored the polynomial \((ax^n + bx^{n-1} + \ldots)\) by \((x-c)\).
5. Correctly interpret answers to the expression \((ax^n + bx^{n-1} + \ldots) \div (x-d)\) with a remainder \(r = n\) as having evaluated \(f(d) = n\) where \(f(x) = (ax^n + bx^{n-1} + \ldots)\). (Note: this is synthetic substitution).
6. Given a known root of polynomial of degree 3 or 4, find the remaining roots using either long division or synthetic division.

Resources:

Houghton-Mifflin; (1990); Algebra & Trigonometry, Structure and Method, pp. 364-371

Relevant Massachusetts Curriculum Framework

12.N.1 – Define complex numbers (e.g., \(a + bi\)) and operations on them, in particular, addition, subtraction, multiplication, and division. Relate the system of complex numbers to the system of real and rational numbers.
12.P.7 – Find solutions to quadratic equations (with real coefficients and real or complex roots) and apply to the solution of problems.
12.P.8 - Solve a variety of equations and inequalities using algebraic, graphical, and numerical methods., including the quadratic formula.; use technology where appropriate. Include polynomial, exponential, logarithmic, and trigonometric functions; expressions involving absolute values; trigonometric relations; and simple rational expressions.
12.P.10 – Use symbolic, numeric, and graphical methods to solve systems of equations algebraic, exponential, and logarithmic expressions. Also use technology where appropriate. Describe the relationships among the methods.
LESSON 1: MENTAL MATH – (5 Minutes)

Start with a quick review of different ways of indicating division. Let’s look at the following; which are equivalent? (Note: On transparency)

1. \( \frac{x^2 + 2x + 3}{x + 1} \)  
   a. \( (y - 2)(y^2 - 4) \)

2. \( \frac{432}{27} \)  
   b. 190:27

3. \( (y^2 - 4) ÷ (y - 2) \)  
   c. \( 27\sqrt{432} \)

4. 190 ÷ 27  
   d. \( x + 1 \) \( x^2 + 2x + 3 \)

Note: 1/d, 3/a, and 4/b match. 2/c are not matches since c contains a radicand, not long division. Forms include: Fraction bars: 1 and 2, Ratio: b, Long Division: a and d, and Algebraic division symbol: 3 and 4.

What is important above is that regardless of how the problem is written, division can be indicated mathematically in a number of forms. In the lessons that we will be doing over the next 2 days, we will use three of these forms, the fraction bar, long division, and Algebraic expressions.

CLASS ACTIVITIES – (Note: 45 Minute Lesson Plan)

Let’s scaffold the teaching of Long Division of polynomials by binomials.

Now, let’s try a few examples of division, do these problems in your notes, and let’s discuss the answers:

Ex 1: \( 14 \) \( \sqrt{287} = 20 R 7, or 20 \frac{7}{14}, or 20 \frac{1}{2} or 41/2 \)

Note, in this example, we can indicate our answer in a number of ways. While all are technically correct, we will be focusing our long division and synthetic division of polynomials on answers that are in simplest form (the last 2 in the solution below).

Ex 2: \( (3x^2 + x) ÷ (x) = \frac{3x^2 + x}{x} = x \sqrt{3x^2 + x} = 3x + 1 \)
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Note: In this example of a polynomial divided by a monomial, we can solve this problem by factoring an “x” out of the numerator. If we do, then we end up with \((3x + 1)\) as the answer. Note however, the equivalence of the three forms of division.

Ex. 3: \(\frac{3x + 4}{x} = 3 + \frac{4}{x}\)

Note: If we perform the long division: \(x)\overline{3x+4}\), we end up with an answer of 3 with a remainder of 4. This can be written as \(3 \text{ R } 4\), or more properly (for Algebra) as \(3 + \frac{4}{x}\).

Ex 4: \((x^3 - 5x^2 + 8x - 4) ÷ (x - 1) = x^2 - 4x + 4\)

Ex 5: \((2x^3 + 4x^2 + 7x + 1) ÷ (x + 1) = x^2 + 2x + 5 \text{ R } -4\)

Notes:

a. In this example, we are performing long division of the polynomial of degree three by a binomial. In doing so, what do we get? \((a \text{ quadratic})\).

b. If we were to factor the quadratic \(x^2 - 4x + 4\), what would we get? \((x - 2)^2\)

c. So now that we have performed the long division of \((x^3 - 5x^2 + 8x - 4)\) by \((x - 1)\) and found that the dividend is \(x^2 - 4x + 4\) with a remainder of zero. But \(x^2 - 4x + 4\) can be written as something else. What is that? \((x - 2)^2\)

d. Therefore, what can we say about the polynomial of degree three \((x^3 - 5x^2 + 8x - 4)\)?

\(That \ its \ factors \ are: \ (x - 2)^2 \times (x-1)\)

e. Finally, when the remainder is zero (R = 0) after division by a binomial, we say that we have found a “zero” of that polynomial. What else can we call a “zero” \((A \ factor)\).
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c. By the way, if we were to say that \( f(x) = (2x^3 + 4x^2 + 7x + 1) \), then what would \( f(-1) \) be equal to? (-4)

d. Notice the remainder above, have we just discovered something new?

Synthetic Division

Now lets try a new method called Synthetic Division. Synthetic Division is an algorithm designed to solve the general case of dividing an arbitrary polynomial of the form

\[ ax^n + bx^{n-1} + cx^{n-2} + \ldots \] by the binomial \((x - c)\). Note that the binomial MUST have a leading coefficient of one (1). Lets do example #4 above \( x - 1 \mid x^3 - 5x^2 + 8x - 4 \) using synthetic division.

\[
\begin{array}{c|cccc}
 & 1 & -5 & 8 & -4 \\
\hline
& & & & \\
\end{array}
\]

We first create a horizontal line. The first number is the constant \( c \) in the binomial \((x - c)\). Notice that since the binomial is defined as \((x - c)\), this has the effect of changing the sign of \( c \). Next, we put the coefficients of the polynomial in horizontal line with the constant. Draw the two lines as I have drawn above. Now, we add the columns, and multiply to create the next number up diagonally. Look at the following. The red lines indicate additions. The blue lines indicate multiplication by the constant of division (in this case 1).

\[
\begin{array}{c|cccc}
1 & 1 & -5 & 8 & -4 \\
\hline
& & & & \\
\end{array}
\]

\[
result \quad 1 -4 \quad 4 \\
= x^2 - 4x + 4 R 0
\]

Ex 4: \( (x^3 - 5x^2 + 8x - 4) \div (x - 1) = x - 1 \)

\[
\begin{array}{r}
\frac{x^3 - x^2}{-4x^2 + 8x} \\
\frac{-4x^2 + 4x}{4x - 4} \\
\frac{4x - 4}{0}
\end{array}
\]

Notice also, that just as in example 4 above, the intermediate results (in green) are the same as in the long division. But instead of subtraction of binomials at each step, we are adding simple numbers. The numbers on the bottom line are the coefficients of the polynomial of degree \( n-1 \). In our example, we had a polynomial of degree three, divided by a binomial of degree one, so we end up with an answer that is a polynomial of degree two, or a quadratic. The last number on the bottom line is the remainder (in this case zero).
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So now try our Example 5 above \((2x^3 + 4x^2 + 7x + 1) \div (x + 1)\) using synthetic division. Remember that the constant of division is in the form of \((x - c)\). In this case, what is “c”? *(negative one)*

\[
\begin{array}{c|cccc}
-1 & 2 & 4 & 7 & 1 \\
\hline
 & -2 & -2 & -5 & \\
result & 2 & 2 & 5 & -4 \\
\end{array}
\]

\[2x^2 + 2x + 5 \text{ R } -4 \text{ or } 2x^2 + 2x - (4/x+1)\]

Great! Since the remainder is non-zero, instead of factoring the polynomial \((2x^3 + 4x^2 + 7x + 1)\), we have done what? *(evaluated it at } x = -1)* When we have a non-zero remainder in synthetic division, we can also call this “Synthetic Substitution”. That is substituting the value “-c” for “x” in the original expression or function.

Let’s do one last problem: (Example 2 from p 280-281 in textbook)

\((x^3 - 13x + 12) \div (x + 4)\)

\[
\begin{array}{c|cccc}
-4 & 1 & 0 & -13 & 12 \\
\hline
 & -4 & 16 & -12 & \\
result & 1 & -4 & 3 & 0 \\
\end{array}
\]

\[x^2 - 4x + 3\]

End of Lesson 1.
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**DAY 1 HOMEWORK:** p282-283: #2-6 column, #9-15 column, and 31.

2) \( \frac{3x - 5}{x + 4} \left( \frac{3x^2 + 7x - 20}{3x^2 + 12x} \right) = \frac{2x^2 + 5x + 2}{-5x - 20} - \frac{5x - 20}{0} \)

4) \( (2x^3 - 3x^2 - 18x - 8) ÷ (x - 4) = x - 4 \left( \frac{2x^3 - 3x^2 - 18x - 8}{2x^3 - 8x^2} \right) = \frac{5x^2 - 18x}{5x^2 - 20x} = \frac{2x - 8}{2x - 8} = 9x - 12 \text{ or } 9x - 12 + \frac{(-32)}{x - 1} \)

6) \( (9x^2 - 21x - 20) ÷ ((x - 1) = x - 1 \left( \frac{9x^2 - 21x - 20}{9x^2 - 9x} \right) = \frac{-12x - 20}{-12x + 12} = \frac{-32}{0} \)

9) \( (x^3 - 4x^2 + 6x - 4) ÷ ((x - 2) = x - 2 \left( \frac{x^2 - 2x + 2}{1} \right) \)

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11) \( (x^3 - 3x^2 - 5x - 25) ÷ ((x - 5) = x^2 + 2x + 5 \)

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13) \( (2x^4 + 6x^3 + 5x^2 - 45) ÷ ((x + 3) = 2x^3 + 5x - 15 \)

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<td>45</td>
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<tr>
<td><strong>result</strong></td>
<td>2</td>
<td>0</td>
<td>5</td>
<td>-15</td>
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15) \((-2x^3 + 5x^2 - x + 2) ÷ (x + 2) = -2x^2 + 9x - 19 R 40\) or \(-2x^2 + 9x - 19 + 40/(x+2)\)

\[
\begin{array}{c|cccc}
-2 & -2 & 5 & -1 & 2 \\
\hline 
\text{result} & -2 & 9 & -19 & 40 \\
\end{array}
\]

31) A polynomial \(P(x)\) is divided by a binomial \(x - a\). The remainder is zero. What conclusion can you draw? Explain.

Answer: An appropriate conclusion is that \((x-a)\) is a factor of the polynomial \(P(x)\). This is true because \((x-a)\) divides “evenly” into \(P(x)\) with a remainder of zero. Another way of saying this is that “\(a\)” is a “zero” of \(P(x)\) or that \(P(a) = 0\).
LESSON 2: MENTAL MATH – (5 Minutes)

Start with quick review of addition and multiplication of complex numbers

We start with a quick review of addition and multiplication of complex numbers. (Note: On transparency)

True or False?

1. \((10 + i) + (-5 + i) = (5 + 2i)\)
2. \((-5 - 2i) - (27 + 3i) = (-32 + i)\)
3. \((10 + i) \times (10 + i) = (100 + i^2)\)
4. \((4 - 2i) \times (4 + 2i) = 20\)
5. \((4 - 3i) \times (2 + 2i) = 14 + 2i\)
6. \((a + bi) \times (c + di) = ac + i(ad + bc) - bd\)

*Note: 1, 4, 5, and 6 are true. 2 and 3 are false.*

This is a quick reminder of how to do addition and multiplication with complex numbers.

CLASS ACTIVITIES – (Note: 45 Minute Lesson Plan)

Let’s start with providing answers to the previous nights homework.

Check to see if there are any problems. Ask to see who needs help on either long division or synthetic division. Provide answers to 2, 4, and 6 below. Answers to 9-15 column and 31 are in the book. See if anyone needs problems worked out. Allow students to go to board to solve.

\[
\begin{align*}
\text{2) } & \quad \frac{3x - 5}{x + 4} \div 3x^2 + 7x - 20 \\
\text{4) } & \quad \frac{2x^2 + 5x + 2}{(2x^3 - 3x^2 - 18x - 8) \div (x - 4)} = x - 4 \div 2x^3 - 3x^2 - 18x - 8 \\
\text{6) } & \quad (9x^2 - 21x - 20) \div ((x - 1) = x - 1 \div 9x^2 - 21x - 20}
\end{align*}
\]
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Now we are going to add something to the previous night’s homework. Let’s look at three specific issues:

1. Synthetic division or substitution when the divisor is in the form of \((ax - b)\) instead of \((x - c)\).
2. Finding “zeroes” of polynomials of degree three when given one zero in the form of \(x = a\).
3. Finding zeroes of polynomials of degree 3 or 4 when given a zero in the form of \(x = a + bi\).

1. Synthetic division or substitution when the divisor is in the form of \((ax - b)\) instead of \((x - c)\).

Let’s now look at another problem. Let’s use synthetic division to solve the following. Look at 7 on your worksheet for tonight.

Prob. 7. \((2x^3 - 3x^2 + 4x - 2) ÷ (2x + 1) = \)

Notice, that the leading coefficient of the binomial \((2x + 1)\) is NOT one. In order to use synthetic division, it must be one, so what do we do? **Wait for someone to come up with idea of dividing the binomial by 2.**

That is right, we will divide the binomial by its leading coefficient. However, when we do that, dividing the polynomial \(P(x) = (2x^3 - 3x^2 + 4x - 2)\) by \((x + \frac{1}{2})\) is NOT the same as dividing by \((2x + 1)\). Does anyone question why? Notice:

\[
\frac{2x^3 - 3x^2 + 4x - 2}{2x + 1} = \frac{1}{2} \times \frac{2x^3 - 3x^2 + 4x - 2}{x + \frac{1}{2}}
\]

So, we will use synthetic division to divide \((2x^3 - 3x^2 + 4x - 2)\) by \((x + \frac{1}{2})\) and then multiply the result by \(\frac{1}{2}\). Correct? Let’s do that.

\[
\begin{array}{cccc}
-\frac{1}{2} & 2 & -3 & 4 & -2 \\
\hline
& 2 & -1 & 2 & -3 \\
\hline
\text{result} & 2 & -4 & 6 & -5
\end{array}
= 2x^2 - 4x + 6 R -5
\]

But, our original equation is not \((2x^3 - 3x^2 + 4x - 2) ÷ (x + \frac{1}{2})\) is it? **No.** Since our original expression is \((2x^3 - 3x^2 + 4x - 2) ÷ (2x + 1)\), what do we have to do with our result? **We have to multiply the result by \(\frac{1}{2}\)**. When we do that, we get? *(see following)*

\[
(2x^3 - 3x^2 + 4x - 2) ÷ (2x + 1) = \frac{1}{2} \times \left(2x^2 - 4x + 6 + \frac{(-5)}{x + \frac{1}{2}}\right) = x^2 - 2x + 3 - \frac{5}{2x + 1}
\]

You will find another problem in your homework tonight like this one.
2. Finding “zeroes” of polynomials of degree three when given one zero in the form of \( x = a \).

Lets move on to another topic. Lets look at problem 21 on page 283 of your textbook. I’d like one of you to read the instructions and the problem:

“One zero is given for each function. Find all of the other zeroes, real or imaginary. Give exact answers.

21. \( f(x) = x^3 - 4x^2 + 2x + 1; \) zero at \( x = 1 \).”

How do you suppose we will solve this problem? *Wait for a student to notice that a zero at \( x = 1 \) means that we can set up a synthetic division to reduce the degree polynomial to a quadratic which can then be solved by factoring or using the quadratic formula.*

That’s right! We can divide the function \( f(x) = x^3 - 4x^2 + 2x + 1 \) by \((x-c)\) where \( c = 1 \). This means we are actually performing the division: \( (x^3 - 4x^2 + 2x + 1) \div (x - 1) \) correct? So let’s do that using either long division or synthetic division:

\[
\begin{array}{cccc}
1 & -4 & 2 & 1 \\
\hline
& 1 & -3 & -1 \\
result & 1 & -3 & -1 & 0 \\
\end{array}
\]

= \( x^2 - 3x - 1 \) Correct?

Now we can factor or use the quadratic formula to solve the resulting quadratic. What are the solutions to \( f(x) = x^3 - 4x^2 + 2x + 1 \)? This is not factorable, so:

\[
a = 1, b=-3, c=-1 \text{ and } \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-1)}}{2(1)} = \frac{3 \pm \sqrt{9 - (-4)}}{2} = \frac{3 \pm \sqrt{13}}{2}
\]

So we have three “zeroes” for this equation. These are: \( x = 1, x = \frac{3 + \sqrt{13}}{2}, \) and \( x = \frac{3 - \sqrt{13}}{2} \).

3. Finding zeroes of polynomials of degree 3 or 4 when given a zero in the form of \( x = a+bi \).

Finally, lets look at solutions to problems where some of the “zeroes” are complex numbers. Let’s look at the following problem:

“One zero is given for each function. Find all of the other zeroes, real or imaginary. Give exact answers.

\( x^4 - 6x^3 + 14x^2 - 16x + 8; \) zero at \( x = (1+i) \).”

If we have a solution to the equation \( x^4 - 6x^3 + 14x^2 - 16x + 8 = 0 \) at \( x = (1+i) \), what else do we know? *Note. This may require some thought. We want the students to recognize that any time there is a complex root, that there is also a conjugate pair. Thus, the complex conjugate root to
x=(1+i) is x=(1-i) Let them think and raise hands when they think they have it. Wait until at least half the class does.

That’s right! Whenever there is one complex “zero” or root, there is another as well. What do we call these pairs of complex roots? *Complex Conjugates.*

So we know that both \((x - (1 + i))\) and \((x - (1 - i))\) are roots to the polynomial expression \(x^4 - 6x^3 + 14x^2 - 16x + 8\). Given that, we have two options, we can either use synthetic division twice with \((x = (1 - i))\) and \((x = (1 - i))\) to figure out the resulting quadratic equation, or we can multiply the two terms \((x - (1 + i)) \times (x - (1 - i))\) to find a quadratic and then use long division to find the remaining quadratic. Lets do the former (synthetic division).

$$
\begin{array}{c|cccc}
\text{1-}\text{i} & 1 & -6 & 14 & -16 \\
\hline & 1-\text{i} & -6+4\text{i} & 12-4\text{i} & -8 \\
\text{result} & 1 & -5-\text{i} & 8+4\text{i} & -4-4\text{i} & 0
\end{array}
$$

\[= x^3 + (-5-\text{i})x^2 + (8+4\text{i})x + (-4-4\text{i})\text{ Correct?}

Now lets do this again by dividing the resulting polynomial of degree three by the complex conjugate of \(x = 1-\text{i}\) which is \(x=1+\text{i}\).

$$
\begin{array}{c|cccc}
\text{1+}\text{i} & 1 & -5-\text{i} & 8+4\text{i} & -4-4\text{i} \\
\hline & 1+\text{i} & -4-4\text{i} & 4+4\text{i} & \\
\text{result} & 1 & -4 & 4 & 0
\end{array}
$$

\[= x^2 - 4x + 4 \text{ Correct?}

What are the factors or zeroes of \(x^2 - 4x + 4\)? *That is correct, this is \((x - 2)^2\).* So now we know that the expression \(x^4 - 6x^3 + 14x^2 - 16x + 8\) has zeroes at \(x = 2\), \(x = (1+i)\), and \(x = (1-i)\).

You will find one problem on your homework like this one. In addition, you are more likely to find problems where the real roots are given, and you end up finding the complex or imaginary roots when using the quadratic equation.

*End of Lesson 2.*
**DAY 2 HOMEWORK**: p283: #19, 23 and attached worksheet (Even Problems and #11).

One zero is given for each function. Find all of the other zeroes, real or imaginary. Give exact answers.

19. \( f(x) = x^3 - x^2 - 6x \); zero at \( x = -2 \).
23. \( f(x) = 2x^3 + x^2 + 1 \); zero at \( x = -1 \)

_Solutions:_

19. \( f(x) = x^3 - x^2 - 6x \); zero at \( x = -2 \).

\[
f(x) = x^3 - x^2 - 6x = x(x^2 - x - 6) = x(x + 2)(x - 3) \therefore \text{zeros at } x=0, x=-2, x=3
\]

23. \( f(x) = 2x^3 + x^2 + 1 \); zero at \( x = -1 \)

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\[
2x^2 - 3x + 1
\]

\[
a = 2, b=-1, c=-1 \text{ and } \frac{-b\pm\sqrt{b^2-4ac}}{2a} = \frac{-(-1)\pm\sqrt{(-1)^2-4(2)(1)}}{2(2)} = \frac{1\pm\sqrt{1-8}}{4} = \frac{1\pm i\sqrt{7}}{4}
\]

Zeroes at: \( x = -1, x=\frac{1+i\sqrt{7}}{4}, \text{ and } x=\frac{1-i\sqrt{7}}{4} \)
Long Division and Synthetic Division of Polynomials – Mr. Ruby

Worksheet – Long Division and Synthetic Division  

Name: ____________________

Part A: Divide Using Long Division

11. \( x + 2 \div x^2 + 3x + 4 \)  
12. \( \frac{6x^4 - 3x^3 + 5x^2 + 2x - 6}{3x^2 - 2} \)

3. \( (x^3 - x^2 - 10x + 10) \div (x - 3) \)  
4. \( \frac{2x^4 - 3x^2 + 7x - 8}{x^2 + x - 3} \)

Part B: Divide Using Synthetic Division

5. \( (2s^3 - 29s + 13) \div (s + 4) \)  
6. \( \frac{y + 2}{y^4 - 4y^2 + y + 4} \)

7. \( (2x^3 - 3x^2 + 4x - 2) \div (2x + 1) \)  
8. \( \frac{4x^3 + 2x^2 - 4x + 3}{2x + 3} \)

9. \( \frac{x^6 - 1}{x + 1} \)  
10. \( \frac{z^3 - 2z^2 + 4z - 5}{z - 2i} \)

Part C: Find the zeroes.

11. One zero is given for the function. Find all of the other zeroes, real or imaginary. Give exact answers. \( f(x) = x^4 - 8x^3 + 25x^2 - 36x + 20 \); zero at \( x = 2 - i \).

Source: Houghton-Mifflin; (1990); Algebra & Trigonometry, Structure and Method, pp. 364-371
Worksheet – Long Division and Synthetic Division

Part A: Divide Using Long Division

1. \(x + 2\) \(\overline{x^2 + 3x + 4} = x + 1 + \frac{2}{x + 1}\)

2. \(\frac{6x^4 - 3x^3 + 5x^2 + 2x - 6}{3x^2 - 2} = 2x^2 - x + 3\)

3. \((x^3 - x^2 - 10x + 10) \div (x - 3) = x^2 + 2x - 4 + \frac{-2}{x - 3}\)

4. \(\frac{2x^4 - 3x^2 + 7x - 8}{x^2 + x - 3} = 2x^2 - 2x + 5 + \frac{-4x + 7}{x^2 + x - 3}\)

Part B: Divide Using Synthetic Division

5. \((2s^3 - 29s + 13) \div (s + 4) = 2s^2 - 8s + 3 + \frac{1}{s + 4}\)

6. \(y + 2\) \(\overline{y^4 - 4y^2 + y + 4} = y^3 - 2y^2 + 1 + \frac{2}{y + 2}\)

7. \((2x^3 - 3x^2 + 4x - 2) \div (2x + 1) = x^2 - 2x + 3 + \frac{-5}{2x + 1}\)

8. \(\frac{4x^3 + 2x^2 - 4x + 3}{2x + 3} = 2x^2 - 2x + 1\)

9. \(\frac{x^6 - 1}{x + 1} = x^5 - x^4 + x^3 - x^2 + x - 1\)

10. \(\frac{z^3 - 2z^2 + 4z - 5}{z - 2i} = z^2 + (-2 + 2i)z - 4i + \frac{3}{z - 2i}\)
Long Division and Synthetic Division of Polynomials – Mr. Ruby

Part C: Find the zeroes.

11. One zero is given for the function. Find all of the other zeroes, real or imaginary. Give exact answers. \( f(x) = x^4 - 8x^3 + 25x^2 - 36x + 20 \); zero at \( x = 2+i \).

\[
\begin{array}{c|cccc}
  2-i & 1 & -8 & 25 & -36 & 20 \\
  \hline
  \text{result} & 1 & -6-i & 12+4i & -8-4i & 0 \\
\end{array}
\]

\[\text{result} = x^3 + (-6-i)x^2 + (12+4i)x + (-8-4i)\]

Now let's do this again by dividing the resulting polynomial of degree three by the complex conjugate of \( x = 2-i \) which is \( x = 2+i \).

\[
\begin{array}{c|cccc}
  2+i & 1 & -6-i & 12+4i & -8-4i \\
  \hline
  \text{result} & 1 & -4 & 4 & 0 \\
\end{array}
\]

\[\text{result} = x^2 - 4x + 4 \quad \text{Correct?}\]

So now we know that the expression \( f(x) = x^4 - 8x^3 + 25x^2 - 36x + 20 \) has zeroes at \( x = 2 \), \( x = (2+i) \), and \( x = (2-i) \).

Source: Houghton-Mifflin; (1990); \textit{Algebra & Trigonometry, Structure and Method}, pp. 364-371