CHAPTER 18

ELECTRICAL PROPERTIES

PROBLEM SOLUTIONS

Ohm's Law
Electrical Conductivity

18.1 This problem calls for us to compute the electrical conductivity and resistance of a silicon specimen.

(a) We use Equations 18.3 and 18.4 for the conductivity, as

$$\sigma = \frac{1}{\rho} = \frac{Il}{VA} = \frac{Il}{V\pi\left(\frac{d}{2}\right)^2}$$

And, incorporating values for the several parameters provided in the problem statement, leads to

$$\sigma = \frac{(0.25\ \text{A})(45\ \text{x}\ 10^{-3}\ \text{m})}{(24\ \text{V})(\pi)\left(\frac{7.0\ \text{x}\ 10^{-3}\ \text{m}}{2}\right)^2} = 12.2\ (\Omega\text{ - m})^{-1}$$

(b) The resistance, *R*, may be computed using Equations 18.2 and 18.4, as

$$R = \frac{l}{\sigma A} = \frac{l}{\sigma\pi\left(\frac{d}{2}\right)^2}$$

$$= \frac{57\ \text{x}\ 10^{-3}\ \text{m}}{\left[12.2\ (\Omega - \text{m})^{-1}\right](\pi)\left(\frac{7.0\ \text{x}\ 10^{-3}\ \text{m}}{2}\right)^2} = 121.4\ \Omega$$

18.3 This problem asks that we compute, for a plain carbon steel wire 3 mm in diameter, the maximum length such that the resistance will not exceed 20 Ω. From Table 18.1 for a plain carbon steel $\sigma = 0.6 \text{ x } 10^7 \ (\Omega\text{-m})^{-1}$. If d is the diameter then, combining Equations 18.2 and 18.4 leads to

$$l = R\sigma A = R\sigma\pi\left(\frac{d}{2}\right)^2$$

$$= (20 \ \Omega)\left[0.6 \ \text{x} \ 10^7 \ (\Omega - \text{m})^{-1}\right](\pi)\left(\frac{3 \ \text{x} \ 10^{-3} \ \text{m}}{2}\right)^2 = 848 \ \text{m}$$

Conduction in Terms of Band and Atomic Bonding Models

18.8 This question asks that we explain the difference in electrical conductivity of metals, semiconductors, and insulators in terms of their electron energy band structures.

For metallic materials, there are vacant electron energy states adjacent to the highest filled state; thus, very little energy is required to excite large numbers of electrons into conducting states. These electrons are those that participate in the conduction process, and, because there are so many of them, metals are good electrical conductors.

There are no empty electron states adjacent to and above filled states for semiconductors and insulators, but rather, an energy band gap across which electrons must be excited in order to participate in the conduction process. Thermal excitation of electrons will occur, and the number of electrons excited will be less than for metals, and will depend on the band gap energy. For semiconductors, the band gap is narrower than for insulators; consequently, at a specific temperature more electrons will be excited for semiconductors, giving rise to higher conductivities.

Electron Mobility

18.9 The drift velocity of a free electron is the average electron velocity in the direction of the force imposed by an electric field.

The mobility is the proportionality constant between the drift velocity and the electric field. It is also a measure of the frequency of scattering events (and is inversely proportional to the frequency of scattering).

18.17 We are asked to select which of several metals may be used for a 3 mm diameter wire to carry 12 A, and have a voltage drop less than 0.01 V per foot (300 mm). Using Equations 18.3 and 18.4, let us determine the minimum conductivity required, and then select from Table 18.1, those metals that have conductivities greater than this value. Combining Equations 18.3 and 18.4, the minimum conductivity is just

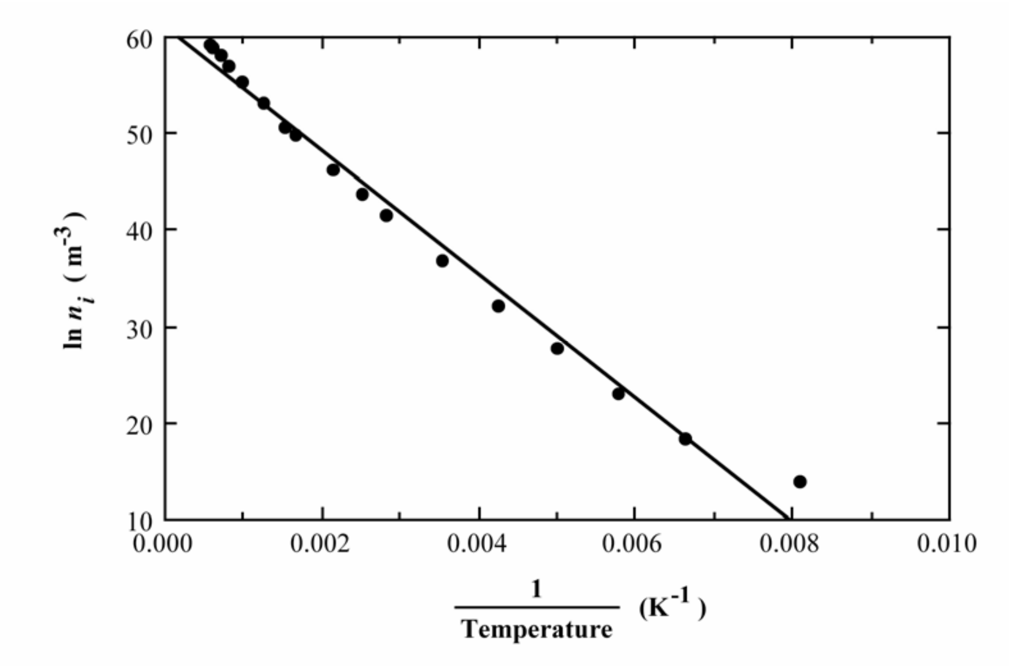
$$\sigma = \frac{Il}{VA} = \frac{Il}{V\pi\left(\frac{d}{2}\right)^2}$$

$$= \frac{(12\ \text{A})(300\ \text{x}\ 10^{-3}\ \text{m})}{(0.01\ \text{V})\ (\pi)\left(\frac{3\ \text{x}\ 10^{-3}\ \text{m}}{2}\right)^2} = 5.1\ \text{x}\ 10^7\ (\Omega\text{ - m})^{-1}$$

Thus, from Table 18.1, only copper, and silver are candidates.

18.19 This problem asks that we make plots of ln n_i versus reciprocal temperature for both Si and Ge, using the data presented in Figure 18.16, and then determine the band gap energy for each material realizing that the slope of the resulting line is equal to – $E_g/2k$.

Below is shown such a plot for Si.



The slope of the line is equal to

$$\text{Slope} = \frac{\Delta \ln \eta_i}{\Delta\left(\frac{1}{T}\right)} = \frac{\ln \eta_1 - \ln \eta_2}{\frac{1}{T_1} - \frac{1}{T_2}}$$

Let us take $1/T_1$ = 0.001 and $1/T_2$ = 0.007; their corresponding ln η values are ln η_1 = 54.80 and ln η_2 = 16.00. Incorporating these values into the above expression leads to a slope of

$$\text{Slope} = \frac{54.80 - 16.00}{0.001 - 0.007} = -6470$$

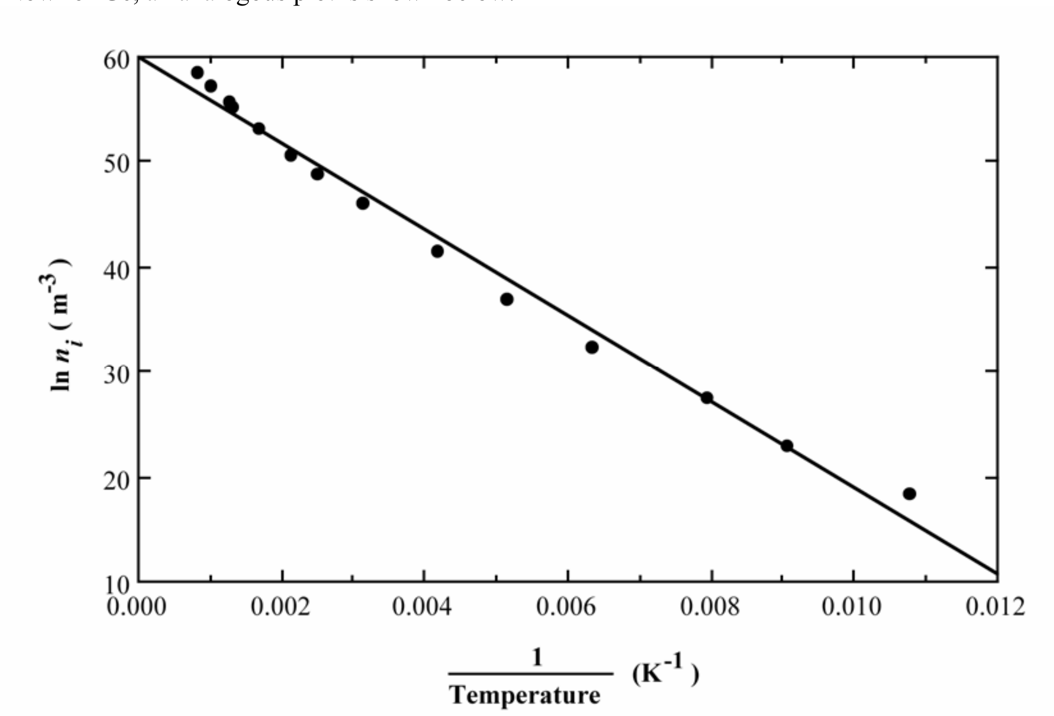
This slope leads to an E_g value of

$$E_g = -\,2k\,(\text{Slope})$$

$$= -2\left(8.62 \text{ x } 10^{-5} \text{ eV/K}\right)(-6470) = 1.115 \text{ eV}$$

The value cited in Table 18.3 is 1.11 eV.

Now for Ge, an analogous plot is shown below.



We calculate the slope and band gap energy values in the manner outlined above. Let us take $1/T_1$ = 0.001 and $1/T_2$ = 0.011; their corresponding ln η values are ln η_1 = 55.56 and ln η_2 = 14.80. Incorporating these values into the above expression leads to a slope of

$$\text{Slope} = \frac{55.56 - 14.80}{0.001 - 0.011} = -4076$$

This slope leads to an E_g value of

$$E_g = -2k\,(\text{Slope})$$

$$= -2\left(8.62 \times 10^{-5}\ \text{eV/K}\right)(-4076) = 0.70\ \text{eV}$$

This value is in good agreement with the 0.67 eV cited in Table 18.3.

Extrinsic Semiconduction

18.24 These semiconductor terms are defined in the Glossary. Examples are as follows: intrinsic--high purity (undoped) Si, GaAs, CdS, etc.; extrinsic--P-doped Ge, B-doped Si, S-doped GaP, etc.; compound--GaAs, InP, CdS, etc.; elemental--Ge and Si.

18.26 The explanations called for are found in Section 18.11.

18.28 Nitrogen will act as a donor in Si. Since it (N) is from group VA of the periodic table (Figure 2.6), and an N atom has one more valence electron than an Si atom.

Boron will act as an acceptor in Ge. Since it (B) is from group IIIA of the periodic table, a B atom has one less valence electron than a Ge atom.

Sulfur will act as a donor in InSb. Since S is from group VIA of the periodic table, it will substitute for Sb; also, an S atom has one more valence electron than an Sb atom.

Indium will act as a donor in CdS. Since In is from group IIIA of the periodic table, it will substitute for Cd; and, an In atom has one more valence electron than a Cd atom.

Arsenic will act as an acceptor in ZnTe. Since As is from group VA of the periodic table, it will substitute for Te; furthermore, an As atom has one less valence electron than a Te atom.

18.36 This question asks that we compare and then explain the difference in temperature dependence of the electrical conductivity for metals and intrinsic semiconductors.

For metals, the temperature dependence is described by Equation 18.10 (and converting from resistivity to conductivity using Equation 18.4), as

$$\sigma = \frac{1}{\rho_0 + aT}$$

That is, the electrical conductivity decreases with increasing temperature.

Alternatively, from Equation 18.8, the conductivity of metals is equal to

$$\sigma = n|e|\mu_e$$

As the temperature rises, n will remain virtually constant, whereas the mobility (μ_e) will decrease, because the thermal scattering of free electrons will become more efficient. Since $|e|$ is independent of temperature, the net result will be diminishment in the magnitude of σ.

For intrinsic semiconductors, the temperature-dependence of conductivity is just the opposite of that for metals—i.e, conductivity increases with rising temperature. One explanation is as follows: Equation 18.15 describes the conductivity; i.e.,

$$\sigma = n|e|(\mu_e + \mu_h) = p|e|(\mu_e + \mu_h)$$
$$= n_i|e|(\mu_e + \mu_h)$$

Both n and p increase dramatically with rising temperature (Figure 18.16), since more thermal energy becomes available for valence band-conduction band electron excitations. The magnitudes of μ_e and μ_h will diminish somewhat with increasing temperature (per the upper curves of Figures 18.19a and 18.19b), as a consequence of the thermal scattering of electrons and holes. However, this reduction of μ_e and μ_h will be overwhelmed by the increase in n and p, with the net result is that σ increases with temperature.

An alternative explanation is as follows: for an intrinsic semiconductor the temperature dependence is represented by an equation of the form of Equation 18.36. This expression contains two terms that involve temperature—a preexponential one (in this case $T^{-3/2}$) and the other in the exponential. With rising temperature the preexponential term decreases, while the exp ($-E_g/2kT$) parameter increases. With regard to relative magnitudes, the exponential term increases much more rapidly than the preexponential one, such that the electrical conductivity of an intrinsic semiconductor increases with rising temperature.