

### Stress-Strain Behavior

6.3 This problem calls for us to calculate the elastic strain that results for a copper specimen stressed in tension. The cross-sectional area is just  $(15.2 \text{ mm}) \times (19.1 \text{ mm}) = 290 \text{ mm}^2 (= 2.90 \times 10^{-4} \text{ m}^2 = 0.45 \text{ in.}^2)$ ; also, the elastic modulus for Cu is given in Table 6.1 as 110 GPa (or  $110 \times 10^9 \text{ N/m}^2$ ). Combining Equations 6.1 and 6.5 and solving for the strain yields

$$\varepsilon = \frac{\sigma}{E} = \frac{F}{A_0 E} = \frac{44,500 \text{ N}}{(2.90 \times 10^{-4} \text{ m}^2)(110 \times 10^9 \text{ N/m}^2)} = 1.39 \times 10^{-3}$$

6.5 This problem asks us to compute the elastic modulus of aluminum. For a square cross-section,  $A_0 = b_0^2$ , where  $b_0$  is the edge length. Combining Equations 6.1, 6.2, and 6.5 and solving for  $E$ , leads to

$$\begin{aligned}
 E &= \frac{\sigma}{\varepsilon} = \frac{\frac{F}{A_0}}{\frac{\Delta l}{l_0}} = \frac{Fl_0}{b_0^2 \Delta l} \\
 &= \frac{(66,700 \text{ N})(125 \times 10^{-3} \text{ m})}{(16.5 \times 10^{-3} \text{ m})^2 (0.43 \times 10^{-3} \text{ m})} \\
 &= 71.2 \times 10^9 \text{ N/m}^2 = 71.2 \text{ GPa} \quad (10.4 \times 10^6 \text{ psi})
 \end{aligned}$$

6.7 (a) This portion of the problem calls for a determination of the maximum load that can be applied without plastic deformation ( $F_y$ ). Taking the yield strength to be 345 MPa, and employment of Equation 6.1 leads to

$$\begin{aligned} F_y &= \sigma_y A_0 = (345 \times 10^6 \text{ N/m}^2)(130 \times 10^{-6} \text{ m}^2) \\ &= 44,850 \text{ N} \quad (10,000 \text{ lb}_f) \end{aligned}$$

(b) The maximum length to which the sample may be deformed without plastic deformation is determined from Equations 6.2 and 6.5 as

$$\begin{aligned} l_i &= l_0 \left( 1 + \frac{\sigma}{E} \right) \\ &= (76 \text{ mm}) \left[ 1 + \frac{345 \text{ MPa}}{103 \times 10^3 \text{ MPa}} \right] = 76.25 \text{ mm} \quad (3.01 \text{ in.}) \end{aligned}$$

6.9 This problem asks that we calculate the elongation  $\Delta l$  of a specimen of steel the stress-strain behavior of which is shown in Figure 6.21. First it becomes necessary to compute the stress when a load of 65,250 N is applied using Equation 6.1 as

$$\sigma = \frac{F}{A_0} = \frac{F}{\pi \left(\frac{d_0}{2}\right)^2} = \frac{65,250 \text{ N}}{\pi \left(\frac{8.5 \times 10^{-3} \text{ m}}{2}\right)^2} = 1150 \text{ MPa (170,000 psi)}$$

Referring to Figure 6.21, at this stress level we are in the elastic region on the stress-strain curve, which corresponds to a strain of 0.0054. Now, utilization of Equation 6.2 to compute the value of  $\Delta l$

$$\Delta l = \varepsilon l_0 = (0.0054)(80 \text{ mm}) = 0.43 \text{ mm (0.017 in.)}$$

### Elastic Properties of Materials

6.14 (a) We are asked, in this portion of the problem, to determine the elongation of a cylindrical specimen of steel. Combining Equations 6.1, 6.2, and 6.5, leads to

$$\sigma = E\varepsilon$$

$$\frac{F}{\pi \left( \frac{d_0^2}{4} \right)} = E \frac{\Delta l}{l_0}$$

Or, solving for  $\Delta l$  (and realizing that  $E = 207$  GPa, Table 6.1), yields

$$\Delta l = \frac{4F l_0}{\pi d_0^2 E}$$

$$= \frac{(4)(48,900 \text{ N})(250 \times 10^{-3} \text{ m})}{(\pi)(15.2 \times 10^{-3} \text{ m})^2 (207 \times 10^9 \text{ N/m}^2)} = 3.25 \times 10^{-4} \text{ m} = 0.325 \text{ mm} \text{ (0.013 in.)}$$

(b) We are now called upon to determine the change in diameter,  $\Delta d$ . Using Equation 6.8

$$\nu = -\frac{\varepsilon_x}{\varepsilon_z} = -\frac{\Delta d / d_0}{\Delta l / l_0}$$

From Table 6.1, for steel,  $\nu = 0.30$ . Now, solving the above expression for  $\Delta d$  yields

$$\Delta d = -\frac{\nu \Delta l d_0}{l_0} = -\frac{(0.30)(0.325 \text{ mm})(15.2 \text{ mm})}{250 \text{ mm}}$$

$$= -5.9 \times 10^{-3} \text{ mm} \text{ } (-2.3 \times 10^{-4} \text{ in.})$$

The diameter will decrease.

6.19 We are asked to ascertain whether or not it is possible to compute, for brass, the magnitude of the load necessary to produce an elongation of 1.9 mm (0.075 in.). It is first necessary to compute the strain at yielding from the yield strength and the elastic modulus, and then the strain experienced by the test specimen. Then, if

$$\varepsilon(\text{test}) < \varepsilon(\text{yield})$$

deformation is elastic, and the load may be computed using Equations 6.1 and 6.5. However, if

$$\varepsilon(\text{test}) > \varepsilon(\text{yield})$$

computation of the load is not possible inasmuch as deformation is plastic and we have neither a stress-strain plot nor a mathematical expression relating plastic stress and strain. We compute these two strain values as

$$\varepsilon(\text{test}) = \frac{\Delta l}{l_0} = \frac{1.9 \text{ mm}}{380 \text{ mm}} = 0.005$$

and

$$\varepsilon(\text{yield}) = \frac{\sigma_y}{E} = \frac{240 \text{ MPa}}{110 \times 10^3 \text{ MPa}} = 0.0022$$

Therefore, computation of the load is *not possible* since  $\varepsilon(\text{test}) > \varepsilon(\text{yield})$ .

6.23 This problem asks that we ascertain which of four metal alloys will not (1) experience plastic deformation, and (2) elongate more than 1.3 mm when a tensile load of 29,000 N is applied. It is first necessary to compute the stress using Equation 6.1; a material to be used for this application must necessarily have a yield strength greater than this value. Thus,

$$\sigma = \frac{F}{A_0} = \frac{29,000 \text{ N}}{\pi \left( \frac{12.7 \times 10^{-3} \text{ m}}{2} \right)^2} = 230 \text{ MPa}$$

Of the metal alloys listed, aluminum, brass and steel have yield strengths greater than this stress.

Next, we must compute the elongation produced in aluminum, brass, and steel using Equations 6.2 and 6.5 in order to determine whether or not this elongation is less than 1.3 mm. For aluminum

$$\Delta l = \frac{\sigma l_0}{E} = \frac{(230 \text{ MPa})(500 \text{ mm})}{70 \times 10^3 \text{ MPa}} = 1.64 \text{ mm}$$

Thus, aluminum is not a candidate.

For brass

$$\Delta l = \frac{\sigma l_0}{E} = \frac{(230 \text{ MPa})(500 \text{ mm})}{100 \times 10^3 \text{ MPa}} = 1.15 \text{ mm}$$

Thus, brass is a candidate. And, for steel

$$\Delta l = \frac{\sigma l_0}{E} = \frac{(230 \text{ MPa})(500 \text{ mm})}{207 \times 10^3 \text{ MPa}} = 0.56 \text{ mm}$$

Therefore, of these four alloys, only brass and steel satisfy the stipulated criteria.

## Tensile Properties

6.24 Using the stress-strain plot for a steel alloy (Figure 6.21), we are asked to determine several of its mechanical characteristics.

(a) The elastic modulus is just the slope of the initial linear portion of the curve; or, from the inset and using Equation 6.10

$$E = \frac{\sigma_2 - \sigma_1}{\varepsilon_2 - \varepsilon_1} = \frac{(1300 - 0) \text{ MPa}}{(6.25 \times 10^{-3} - 0)} = 210 \times 10^3 \text{ MPa} = 210 \text{ GPa} \quad (30.5 \times 10^6 \text{ psi})$$

The value given in Table 6.1 is 207 GPa.

(b) The proportional limit is the stress level at which linearity of the stress-strain curve ends, which is approximately 1370 MPa (200,000 psi).

(c) The 0.002 strain offset line intersects the stress-strain curve at approximately 1570 MPa (228,000 psi).

(d) The tensile strength (the maximum on the curve) is approximately 1970 MPa (285,000 psi).