



Structurally Stable Output Regulation of Nonlinear Systems*

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Necessary and sufficient conditions for the existence of a controller yielding output regulation insensitive to parametric uncertainties are derived and a design method is proposed.

Key Words—Regulator theory; robust control; asymptotic analysis; nonlinear systems; tracking systems.

Abstract—We address a number of issues that were left open in earlier works on the subject of output regulation of nonlinear systems, and we describe an approach to structurally stable regulation that unifies and extends a number of existing results. Moreover, we also address in part the issue of robust regulation, i.e. the issue of achieving regulation in the presence of parameter uncertainties ranging within a prescribed set. © 1997 Elsevier Science Ltd.

1. INTRODUCTION

The problem of controlling the output of a system so as to achieve asymptotic tracking of prescribed trajectories and/or asymptotic rejection of undesired disturbances is a central problem in control theory. A classical set up in which the problem was posed and successfully addressed—in the context of linear, time-invariant and finite-dimensional systems—is the one in which the exogenous inputs, namely commands and disturbances, may range over the set of all possible trajectories of a given autonomous linear, time-invariant and finite-dimensional system, commonly known as the *exosystem*. The most relevant feature of posing the problem in these terms is that, by incorporating a suitable *internal model* of the exosystem into the compensator that provides the control action, asymptotic tracking with closed-loop stability can be achieved, even in the presence of small *variations in certain systems parameters*, on the basis of a relatively *restricted amount of information* about the controlled

plant. The latter, in fact, usually consists in the actual value of the *tracking error* alone, and does not include explicit access to either the actual trajectory to be tracked or to the disturbance to be rejected. An exhaustive presentation of the theory of output regulation for linear, time-invariant and finite-dimensional systems can be found in the works of Davison (1976), Francis (1977) and Francis and Wonham (1976). In particular, Davison (1976) has shown that a compensator that solves the problem can always be viewed as the *interconnection* of two components, called the *servocompensator* and the *stabilizing compensator*, whose roles are respectively to generate the control inputs needed to impose the prescribed asymptotic behavior and to stabilize the resulting closed loop.

The study of the corresponding design problem for *nonlinear*, time-invariant and finite-dimensional systems was initiated—to the best of our knowledge—in Francis and Wonham (1976) and in the subsequent work of Hepburn and Wonham (1984a, b). In particular, Francis and Wonham (1976) and Hepburn and Wonham (1984b) have addressed the problem of regulation in the presence of *constant* exogenous inputs (i.e. set-point control under constant disturbances), and have shown that the incorporation of an internal model into the compensator (i.e. integral control) suffices to guarantee output regulation in the presence of small parameter variations (i.e. a *structurally stable design*, in the terminology of Francis (1977) and Hepburn and Wonham (1984b)). The latter, among other relevant contributions, has shown that any compensator that solves the problem of output regulation in the case of pure error feedback necessarily incorporates an internal model of the exosystem. Further contributions to the problem

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