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THE CONTROLLED CENTER DYNAMICS*

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Abstract. The center manifold theorem is a model reduction technique for determining the local asymptotic stability of an equilibrium of a dynamical system when its linear part is not hyperbolic. The overall system is asymptotically stable if and only if the center manifold dynamics is asymptotically stable. This allows for a substantial reduction in the dimension of the system whose asymptotic stability must be checked. Moreover, the center manifold and its dynamics need not be computed exactly; frequently, a low degree approximation is sufficient to determine its stability. The controlled center dynamics plays a similar role in determining local stabilizability of an equilibrium of a control system when its linear part is not stabilizable. It is a reduced order control system with a pseudoinput to be chosen in order to stabilize it. If this is successful, then the overall control system is locally stabilizable to the equilibrium. Again, usually low degree approximation suffices.

Key words. nonlinear systems, control bifurcations, center manifold theorem

AMS subject classifications. 93C10, 93C15, 37L10, 37N35

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1. Introduction.

Center manifold theory plays an important role in the study of the stability of dynamical systems when the equilibrium point is not hyperbolic. The center manifold is an invariant manifold of the differential equation which is tangent at the equilibrium point to the eigenspace of the neutrally stable eigenvalues. For instance, as the local dynamic behavior "transverse" to the center manifold is relatively simple since it is the one of the flows in the local stable (and unstable) manifolds, the center manifold method isolates the complicated asymptotic behavior by locating an invariant manifold tangent to the subspace spanned by the eigenspace of eigenvalues on the imaginary axis. In practice, one does not compute the center manifold and its dynamics exactly, since this requires the resolution of a quasi-linear PDE which is not easily solvable. In most cases of interest, an approximation of degree two or three of the solution is sufficient. Then we determine the reduced dynamics on the center manifold, study its stability, and conclude about the stability of the original system [24, 26, 21, 6, 15].

The combination of this theory with the normal form approach of Poincaré [25] was used extensively to study parameterized dynamical systems exhibiting bifurcations [27]. The center manifold theorem provides, in this case, a means of systematically reducing the dimension of the state spaces which need to be considered when analyzing bifurcations of a given type. In fact, after determining the center manifold, the analysis of these parameterized dynamical systems is based only on the restriction of the original system on the center manifold whose stability properties are the same as the ones of the full order system.

This approach was also adopted in control theory. The combination of the normal form approach for control systems [20] and center manifold theory enabled the analysis and stabilization of systems with one or two uncontrollable modes in continuous and

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