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BIFURCATION CONTROL VIA STATE FEEDBACK FOR SYSTEMS WITH A SINGLE UNCONTROLLABLE MODE*

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Abstract. The state feedback control of bifurcations with quadratic or cubic degeneracy is addressed for systems with a single uncontrollable mode. Based on normal forms and invariants, the classification of bifurcations for systems with a single uncontrollable mode is obtained (Table 1). Using invariants, stability characterizations are derived for a family of bifurcations, including saddle-node bifurcations, transcritical bifurcations, pitchfork bifurcations, and bifurcations with a cusp or hysteresis phenomenon. Bifurcations in systems under perturbed feedbacks are also addressed. In the case of a saddle-node bifurcation, continuous but not differentiable feedbacks are introduced to locally remove the bifurcation and to achieve the stability.

Key words. bifurcation, invariants, normal form, feedback control

AMS subject classifications. 93C10, 93C15

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1. Introduction.

Nonlinear dynamical systems exhibit complicated performance around bifurcation points. As the parameter of a system is varied, changes may occur in the qualitative structure of its solutions around a point of bifurcation. Using a feedback to stabilize a system with bifurcations has been studied by many authors (see, for instance, [1], [5], [8], [11], [20], and [25]). Many engineering applications of bifurcation control can be found in the literature (e.g., control of surge and rotating stall in engine compressors, flight control under high angle-of-attack). Quadratic and cubic feedbacks were introduced in [1] for the stabilization of bifurcated equilibria. It was proved in [1] that the periodic solution of a Hopf bifurcation can be stabilized by using state feedbacks. For the period doubling bifurcation, the method of harmonic balance was introduced in [8]. A feedback design method for delaying and stabilizing period doubling bifurcations was obtained. In [25], control laws were designed for the suppression of chaos in a thermal convection system model. A review of bifurcation and chaos in control systems can be found in [5]. More references on related topics can be found in [4], a bibliography of publications on bifurcation and chaos in control systems.

The main goal of this paper is to develop a framework for the analysis and control of bifurcations. Stability characterizations are obtained for control systems around bifurcations with quadratic or cubic degeneracy. Several well-known static bifurcations are addressed in this paper in a unified approach. This is made possible by using normal forms. The main results in sections 4 and 5 are summarized in Table 1, which is a complete classification of bifurcations with quadratic or cubic degeneracy for systems having a single uncontrollable mode. Because the system has only one uncontrollable mode, it does not have the Hopf bifurcation if the feedback stabilizes the controllable part. The Hopf bifurcation occurs for normal forms with two uncontrollable modes (see [7]).

What makes this paper unique is the approach based on the normal form and the

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