SIAM J. CONTROL OPTIM.
Vol. 36, No. 1, pp. 193–212, January 1998

011

BIFURCATION AND NORMAL FORM OF NONLINEAR CONTROL SYSTEMS, PART I*

WEI KANG†

Abstract. The bifurcations of control systems with a single input are studied. Based on the normal forms of control systems, the equilibrium sets are classified. A set of quadratic invariants for control systems is found. Sufficient conditions for a system to be linearly controllable or stabilizable near a bifurcation point are given in terms of the quadratic invariants.

Key words. nonlinear systems, bifurcations, normal forms, invariants, linearly controllable, stabilizable

AMS subject classifications. 93C10, 93C15

PII. S0363012995290288

1. Introduction.

Bifurcation theory studies the changes in qualitative structure of the flow of a dynamic system as parameters are varied. Local bifurcation theory focuses on the stability of the bifurcating solution [6], [15]. In this paper, some bifurcation problems for control systems are addressed. Given a control system with parameters and control inputs, the location of the equilibrium points depends on the values of the parameters and control inputs. The set of equilibrium points is not necessarily a smooth manifold in the state and parameter space. Furthermore, the fundamental properties such as stabilizability and controllability change as the equilibrium point is varied. Understanding the change of these properties is important in feedback design. For instance, a bifurcation occurs in the system of axial flow compressor (see [13]). On one branch of the equilibria, the system is linearly controllable. On the other branch, the system is not stabilizable on one side of the bifurcation point. In fact, feedback is found to achieve the desired stability pattern on the controllable branch [12], [11]. The information about controllability and stabilizability along a set of equilibrium points is also helpful when gain scheduling methods are applied to a nonlinear system. In this paper, local bifurcations of equilibrium sets are classified based on the normal forms of control systems. The controllability and stabilizability at points in the equilibrium set depend on the values of the quadratic invariants. Recent research shows that the bifurcation in the Moore and Greitzer model of the axial flow compressor is equivalent to the two branch bifurcation given in Theorems 3.2 and 4.2.

The behavior of any Hopf bifurcation can be reduced to a few different cases. This is possible because a nonlinear dynamic system can be transformed into a simplified normal form based on Poincaré's theory. Since the bifurcation phenomenon is invariant under change of coordinates, one can study the bifurcation of dynamic systems by focusing on their normal forms. This idea simplifies the problem. An affine control system consists of two vector fields (the drift vector and the control vector). To study the bifurcations of control systems, it is necessary to find normal forms in which both vectors are simplified. For linear systems, a normal form is the controller form. In

*Received by the editors August 14, 1995; accepted for publication (in revised form) October 31, 1996. Research supported in part by AFOSR-95-1-0169.
http://www.siam.org/journals/sicon/36-1/29028.html

†Department of Mathematics, Naval Postgraduate School, Monterey, CA 93943 (wkang@math.nps.navy.mil).