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BIFURCATION AND NORMAL FORM OF NONLINEAR CONTROL SYSTEMS, PART II*

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Abstract. The normal forms and invariants of control systems with a parameter are found. Bifurcations of equilibrium sets are classified. The changes of properties such as controllability of the linearization or stabilizability near a bifurcation point of a control system are studied.

Key words. nonlinear systems, bifurcations, normal forms, invariants, linearly controllable, stabilizable

AMS subject classifications. 93C10, 93C15

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1. Introduction. In this paper, we continue the study of bifurcation problems formulated in Part I [10]. We focus on systems with a parameter. Comparing with the results in Part I, systems with a parameter have three types of normal forms instead of two. Another difference is that, in the presence of a parameter, the equilibrium sets are not curves. In fact, they are two-dimensional surfaces in the state-parameter space. In general, there always exists a curve on the equilibrium set such that the system is not linearly controllable at any point on it.

The following nonlinear system with parameter μ is considered:

$$\dot{\xi} = f(\xi, \mu) + g(\xi, \mu)v. \tag{1.1}$$

The variable $\xi \in \mathbb{R}^n$ is the state, $v \in \mathbb{R}$ is the input variable, and the parameter is $\mu \in \mathbb{R}$. The vector fields $f(\xi, \mu)$ and $g(\xi, \mu)$ are assumed to be C^k for some sufficiently large k. Our attention is focused on local bifurcation near the origin $(\xi, \mu) = (0, 0)$. Assume

$$f(0, 0) = 0, \quad g(0, 0) \neq 0.$$

Following Part I, the equilibrium set E is defined to be

$$E = \{(x, \mu) | \exists v_0 \text{ such that } f(x, \mu) + g(x, \mu)v_0 = 0\}. \tag{1.2}$$

The linearization of the system at the origin is (A, B)

$$A = \frac{\partial f}{\partial x}(0, 0), \quad B = g(0, 0).$$

If the system is linearly controllable at $\xi = 0$, $\mu = 0$, then the system is linearly controllable for all equilibrium points in E near $(x, \mu) = (0, 0)$. Therefore, Questions 1–3 formulated in Part I are interesting only if (1.1) is not linearly controllable at the origin. Similar to Part I, we always assume

Assumption.

$$\operatorname{rank}(\begin{bmatrix} B & AB & A^2B & \cdots & A^{n-1}B \end{bmatrix}) = n - 1. \tag{1.3}$$

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