From *SIAM News*, Volume 40, Number 7, September 2007

Pseudospectral Optimal Control Theory Makes Debut Flight, Saves NASA $1M in Under Three Hours

By Wei Kang and Naz Bedrossian

If you think today's gas prices are high, consider the fuel used by spacecraft, at about $10,000/pound. The obvious need to use spacecraft fuel sparingly puts a recent achievement into perspective: On March 3, 2007, by tracking an attitude trajectory developed with pseudospectral optimal control theory, the International Space Station completed a 180-degree maneuver without using any propellant!

At nearly 500,000 pounds and approximately 150 feet by 200 feet, the ISS is massive. To maintain its orbit and perform necessary attitude maneuvers, it is equipped with thrusters and gyroscopes, i.e., spinning wheel momentum-storage devices. Small attitude adjustments can be accomplished with gyroscopes, which are driven by renewable solar energy. Large-angle rotations carried out with the flight software, however, require more momentum than gyroscopes can provide; unless thrusters are used, attitude control is lost. But thrusters have significant downsides: consumption of precious propellant and the risk of solar-array contamination.

How, then, was the ISS maneuvered without propellant? The key was to realize that the cost of performing a rotation depends directly on the specific trajectory followed and that, by using optimal control theory, the trajectory can be shaped to minimize fuel use.

The ISS flight software uses an eigenaxis trajectory, which is the shortest-distance kinematic path. Most spacecraft use this approach, as it is simple to implement in flight software. According to Euler's rotation theorem, any two orientations are related by a common axis, the eigenaxis, about which rotation by a specific angle, the eigen-angle, is possible. To follow the eigenaxis trajectory, the attitude control system must overcome environmental dynamics, such as torques due to gravity or aerodynamics. As a result, gyroscopes reach momentum cap-acity even if maneuvering at a slower rate.

An alternative to an eigenaxis rotation is a "longer" path that takes advantage of these dynamics to maintain the gyroscopes within their limits, thereby avoiding propellant use. What makes this possible is the attitude-dependence of the environmental torques acting on the ISS. If we think of the ISS as a large orbiting dumbbell, the end nearest the Earth experiences a stronger gravitational pull. When the dumbbell is rotated relative to the orbit radius vector, the differential gravity force creates a moment about its center of mass. The aerodynamic drag force, which depends on the angle between the spacecraft and the airstream, also creates a torque. Control of environmentally induced torques can thus be achieved by modifying the orientation of the ISS. The key problem is how to coordinate and modulate these torques while reaching the desired target without overloading the gyroscopes.

This longer attitude trajectory can be generated by formulating the ISS maneuver as an optimal control problem. Despite considerable progress in modern optimal control theory, solving this nonlinear problem is quite difficult. For instance, a feedback control law requires a solution to the famously difficult Hamilton–Jacobi–Bellman equation. Analytic solutions can rarely be found for systems with nonlinear dynamics, and numerical approximations suffer from the well-known curse of dimensionality. A practical alternative—and the one used for the ISS maneuver described here—is to combine an inner-loop stabilizing controller designed by conventional methods with an outer loop that generates optimal trajectories. A critical challenge in this two-layer control architecture is a reliable outer loop that generates the optimal reference trajectory in a sufficiently short time. Both reliability and computational speed can be mapped to convergence properties of the computational method.

Over the last decade, pseudospectral (PS) techniques have emerged as one of the leading computational methods for solving optimal control problems. To generate and implement the optimal ISS maneuver, the Legendre PS method was used. With a PS method, the original optimal control problem is discretized to formulate a nonlinear programming problem, which is then solved numerically for approximate optimal solutions. PS methods differ from other computational methods in their special discretization at carefully selected nodes. For ex-ample, the so-called Legendre–Gauss–Lobatto (LGL) nodes consist of the points $t_0 = -1$, $t_N = 1$, and t_k, being the roots of $d/dt\, L_N(t)$ in the interval $[-1, 1]$ for $k = 1, 2 \cdots, N-1$, where $L_N(t)$ is the Nth-order Legendre polynomial. The state and control functions are approximated by Nth-order Lagrange polynomials based on interpolation at these nodes.

Existing literature in approximation theory shows that PS methods are easy and accurate methods for the approximations of smooth functions, integrations, and differentiations that are critical to optimal control problems. For differentiation, the derivatives of the state functions at the LGL nodes are easily computed with the differentiation matrix. Thus, the differential equation of the control system is approximated by algebraic equations at the LGL nodes. The integration in the cost functional of an optimal control problem is approximated by the Gauss–Lobatto integration rule. In addition, it is straightforward to discretize the state-control constraints. The collective application of these approximation methods to an optimal control problem results in a nonlinear programming problem with algebraic constraints. PS methods are able to approximate continuous functions with relatively smaller numbers of nodes, thus resulting in lower-dimensional nonlinear programming problems. Given any real-valued function $f(t)$: $[a,b] \rightarrow \Re$, it is well known in approximation theory that sophisticated node selection can lead to very accurate interpolation. For instance, the polynomial interpolation $I_N(f)$ at the LGL nodes has an error

$$\| f(t) - I_N(f)(t) \|_\infty < \frac{C}{N^m}(1 + \Lambda_N),$$

where m is the smoothness of $f(t)$, Λ_N is the Lebesgue constant, and C is a constant [4]. If $f(t)$ is C^∞, then $I_N(f)$ converges at a spectral rate—i.e., faster than any given polynomial rate. This is an extremely fast convergence rate.

Such efficiencies are responsible in part for the increasing popularity of PS methods since the mid-1990s. Subsequent research has revealed several appealing mathematical properties that are important for real-world applications. Whereas the Karush–Kuhn–Tucker (KKT) conditions, rather than Pontryagin's minimum principle (PMP), are used for solving PS optimal control problems, a mapping exists between co-vectors from the KKT conditions and the co-state from PMP [1,2]. The co-vector mapping theorem facilitates quick and easy verification and validation of the computed solution by direct application of the necessary conditions generated from PMP. The reliability issue mentioned earlier is to guarantee that the associated nonlinear programming problem is feasible, and that the approximated solution converges to the true continuous-time optimal trajectory. While this is still an open research area, some key theorems on feasibility and convergence are available to guarantee the convergence of the Legendre PS method [3]. These theorems are particularly important in the solution of industrial-strength problems, in which safety and robustness are crucial for successful implementation. An implementation of the Leg-endre PS optimal control method is available as DIDO [5], a MATLAB optimization package named for Dido, the Queen of Carthage who also gave her name to the isoperimetric problem.

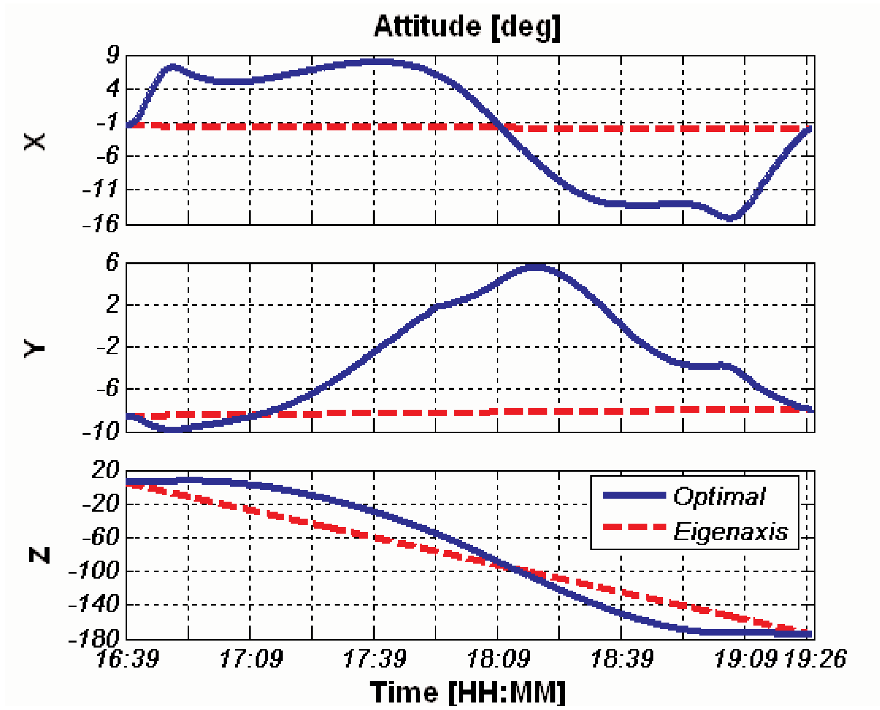

Figure 1. *Optimal versus eigenaxis attitude trajectory.*

The zero propellant maneuver (ZPM) trajectory, developed by the second author and Sagar Bhatt of Draper Laboratory in Hous-ton, uses gyroscopes to rotate the vehicle, without propellant and without modification of flight software. The ZPM is a series of scheduled attitude and rate commands generated by the DIDO-solution to the ISS optimal control problem. The ZPM solves a momentum minimax problem along the trajectory, thereby avoiding capacity limits while meeting attitude, rate, and momentum boundary conditions. Because the trajectory is dependent on accurate knowledge of model parameters, a path that is less sensitive to model uncertainty is preferred over a path with a lower peak momentum. The penalty with such an approach is that the maneuver must be performed at a specified rate, which dictates maneuver completion time. This may not always be feasible, as complex ISS operations are tightly scheduled; in such cases, thrusters must be used.

On March 3, 2007, the ISS was rotated 180 degrees with ZPM in approximately 2 hours and 45 minutes. This was the second success for ZPM, the first being a 90-degree *z*-axis rotation on November 5, 2006. The DIDO-solutions for ZPM were obtained, on average, in 6 hours of computation time on a typical desktop PC. Given performance at this level, near real-time trajectory modifications become feasible. On November 4, for example, payload repositioning on the ISS required a modified 90-degree ZPM trajectory, which was obtained in about 4 hours and uploaded to the ISS the same day. For the 180-degree ZPM, the trajectory was uploaded to the ISS as 80 attitude and rate command pairs spaced 125 seconds apart. The flight-attitude trajectory and, for comparison, an eigenaxis path over the same time frame are shown in Figure 1; the attitude deviation of the ZPM trajectory from an eigenaxis path is clearly evident. In fact, the cumulative ZPM rotation angle was approximately 210 degrees. Peak momentum was less than 76% of capacity. Figure 2 shows animation stills of the maneuver flight data, starting in image 1 and finishing in image 4. Videos of the 90- and 180-degree

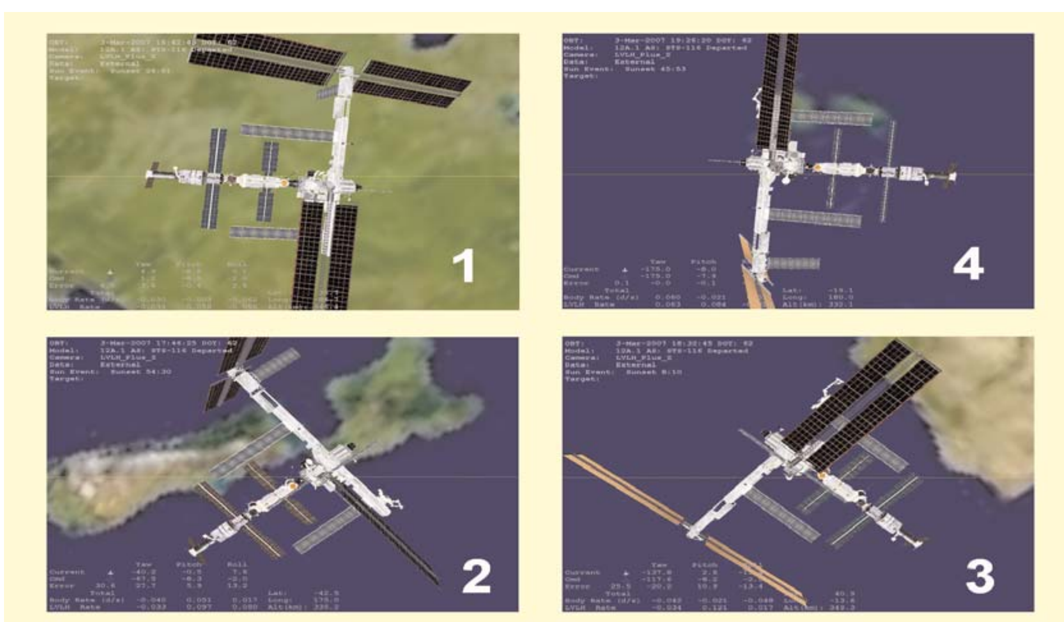

Figure 2. *Animation of flight data for the optimal trajectory.*

ZPMs can be downloaded from http://www.jsc.draper.com/~naz/zpm.html.

According to Louis Nguyen, NASA ISS GN&C systems deputy manager, "The same maneuver performed on January 2, 2007, with thrusters took 1 hour and consumed over 100 pounds of propellant. Using an estimated cost–to–orbit of $10,000/pound, the ZPM saved more than $1,000,000."

The ISS flight application demonstrates the operational power of PS optimal control methods to solve complex, real-world, in-dustrial-strength problems. PS methods, which are currently experiencing rapid development, have also been applied to other optimal control problems and are expected to have even broader impact in years to come.

References

[1] F. Fahroo and I.M. Ross, *Costate estimation by a Legendre pseudospectral method*, Proceedings of the AIAA Guidance, Navigation, and Control Conference, August 10–12, 1998, Boston, MA; J. Guidance, Contr., Dynam., 24:2 (2001), 270–277.

[2] Q. Gong, M. Ross, W. Kang, and F. Fahroo, *Connections between the covector mapping theorem and convergence of pseudo-spectral methods for optimal control*, J. Comput. Optim. Appl., to appear.

[3] Q. Gong, W. Kang, and I.M. Ross, *A pseudo-spectral method for the optimal control of constrained feedback linearizable systems*, IEEE Trans. Automat. Cont., 51:7 (2006), 1115–1129.

[4] J. Hesthaven, S. Gottlieb, and D. Gottlieb, *Spectral Methods for Time-Dependent Problems*, Cambridge University Press, Cambridge, UK, 2007.

[5] I.M. Ross, *User's Manual for DIDO: A MATLAB Application Package for Solving Optimal Control Problems*, NPS Tech. Rep. MAE–03–005, September 2003, Naval Postgraduate School, Monterey, CA.

Wei Kang is a professor of applied mathematics at the Naval Postgraduate School. Naz Bedrossian is group leader for manned space systems at the Charles Stark Draper Laboratory.