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LOCALLY CONVERGENT NONLINEAR OBSERVERS*

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Abstract. We introduce a new method for the design of observers for nonlinear systems using backstepping. The method is applicable to a class of nonlinear systems slighter larger than those treated by Gauthier, Hammouri, and Othman [*IEEE Trans. Automat. Control,* 27 (1992), pp. 875–880]. They presented an observer design method that is globally convergent using high gain. In contrast to theirs, our observer is not high gain, but it is only locally convergent. If the initial estimation error is not too large, then the estimation error goes to zero exponentially. A design algorithm is presented.

Key words. nonlinear estimation, nonlinear observers

AMS subject classifications. 93C10, 93B50, 93E11

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1. Introduction.

The problem of estimating the state of a dynamical system from partial and possibly noisy measurements has a long history. In its nonlinear state space form, one assumes that the dynamics satisfies a known nonlinear differential equation with unknown initial condition and the measurement is a known nonlinear function of the state

$$\begin{aligned} \dot{x} &= f(x), \\ x(0) &= x^0, \\ y &= h(x). \end{aligned} \tag{1.1}$$

The linear form of the problem is

$$\begin{aligned} \dot{x} &= Ax, \\ x(0) &= x^0, \\ y &= Cx. \end{aligned} \tag{1.2}$$

One is given an estimate $\hat{x}^0$ of x^0 and the observations $y(s),\ 0 \le s \le t$, up to time t. The problem is to generate an estimate $\hat{x}(t)$ of $x(t)$ in real time, as the process evolves. The estimate should converge to the true state as $t \to \infty$. Ideally the estimation process should be robust to noise both in the dynamics and in the observations, to the initial state error, and also to modeling errors in the functions $f,\ h$. Furthermore, the error should converge to zero quickly.

One way of approaching this problem is to assume that the dynamics, the initial condition, and the observations are corrupted by noises with known distributions and then to find the conditional density of the state given the past observations. If the dynamics and observations are linear functions of the state and if the noises and

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