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ANALYSIS AND CONTROL OF HOPF BIFURCATIONS*

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Abstract. In this paper, control systems with two uncontrollable modes on the imaginary axis are studied. The main contributions include the local orientation control of periodic solutions and center manifolds, the quadratic normal form of systems with two imaginary uncontrollable modes, the stabilization of the Hopf bifurcation by state feedback, and the quadratic invariants that characterize the nonlinearity of a system and its Hopf bifurcation.

Key words. nonlinear systems, normal forms, quadratic invariants, Hopf bifurcation, center manifold, control

AMS subject classifications. 93C10, 93C15, 37L10, 37N35

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1. Introduction.

Nonlinear dynamical systems exhibit complicated performance around bifurcation points. As the parameter of a system is varied, changes may occur in the qualitative structure of its solutions around a point of bifurcation. Using a feedback to stabilize a system with a bifurcation has been studied by many authors (see, for instance, [1], [2], [7], [10], [12], [21], and [23]). Bifurcation phenomenon appears in a large family of engineering systems. The control of bifurcations has attracted increasing attention in recent years, motivated by engineering applications such as the control of surge and rotating stall in engine compressors [20], [21], and the control of voltage instabilities and collapse [25]. It is known that bifurcations in a linearly controllable system can be delayed or stabilized by a linear feedback [1], [8]. However, nonlinear feedback is essential for systems with uncontrollable bifurcation modes. The authors of [3], [1], [2], [5], [9], and [12] studied the bifurcation and/or stability of systems with one and two uncontrollable modes. Using normal forms, the author of [16], [17], [18] developed the analysis and control design algorithm for systems with one uncontrollable mode.

In this paper, control systems with two uncontrollable modes on the imaginary axis are studied. The main contributions include the local orientation control of periodic solutions and center manifolds, the quadratic normal form of systems with the Hopf bifurcation, the stabilization of the Hopf bifurcation by state feedback, and the quadratic invariants that characterize the nonlinearity of a system and its Hopf bifurcation. In this paper, "bifurcation control" means the control of the local orientation of the periodic solution and the stabilization of the Hopf bifurcation. The results in this paper focus on local bifurcations. Global bifurcations are not addressed here.

The Hopf bifurcation studied in this paper occurs in the Moore–Greitzer model of axial flow engine compressors. A simple version of this model is a four-dimensional ordinary differential equation with nonlinear dynamics exhibiting several kinds of

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