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EXTENDED QUADRATIC CONTROLLER NORMAL FORM AND DYNAMIC STATE FEEDBACK LINEARIZATION OF NONLINEAR SYSTEMS*

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Abstract. In this paper, a set of extended quadratic controller normal forms of linearly controllable nonlinear systems is given, which is the generalization of the Brunovsky form of linear systems. A set of invariants under the quadratic changes of coordinates and feedbacks is found. It is then proved that any linearly controllable nonlinear system is linearizable to second degree by a dynamic state feedback.

Key words. nonlinear systems, quadratic normal forms, invariants, dynamic state feedbacks

AMS(MOS) subject classifications. 93C10, 93C15

1. Introduction. It is well known that there are four normal forms of linear systems: controllable, observable, controller, and observer form. The nonlinear generalizations of these four linear normal forms were given and discussed in Krener [12], Hunt and Su [5], Jakubczyk and Respondek [8], Brockett [1], and Sommer [16], among others. For a system in controller normal form, the design of a stabilizing state feedback control law is a straightforward task. Unfortunately, most controllable systems do not admit a controller normal form, and even when one does, the transformation of a system into controller normal form involves solving a system of first-order partial differential equations (PDEs), which numerically can be quite difficult. For these reasons, the approximate versions of nonlinear controller and observer normal forms were introduced in Krener [11], Krener et al. [13], Phelps and Krener [14], and Karahan [10], among others. It was proved that for certain kinds of nonlinear controllable systems, we can find a nonlinear change of coordinates and nonlinear state feedback that transforms the system into the linear approximation of the plant dynamics, which is accurate to second or higher degree. The computation of such a change of coordinates and state feedback is reduced to solving a set of linear equations. However, these linear equations are not always solvable, and most of the nonlinear systems do not admit such a linear approximation.

In this paper, a set of extended quadratic controller normal forms of linearly controllable systems with single input is given (Theorems 2 and 3). We can consider these normal forms as the extension of the Brunovsky form to the nonlinear systems. Then we prove that, given a nonlinear system, there exists a dynamic state feedback so that the extended system has a linear approximation that is accurate to at least second degree (Theorem 4). This means that any linearly controllable nonlinear system is linearizable to second degree by a dynamic state feedback (see the corollaries).

In this paper, we only consider the single-input systems. The generalization to multi-input systems will be given in another paper.

2. Extended quadratic controller form and dynamic state feedback linearization. From Brunovsky [2] (see also Kailath [9]), we know that any controllable linear system can be transformed into a controller form by a linear change of coordinates. If, in addition, we also allow linear change of coordinates in the input space and linear state

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