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Moving Horizon Numerical Observers of Nonlinear Control Systems

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***Abstract*—In this note, we develop moving horizon numerical observers and analyze the error. In the error estimation, we take into consideration both the integration error and the optimization error. The design facilitates the use of a variety of numerical algorithms to form different observers. As a special case, an Euler-Newton observer is introduced. The numerical observer is independent of any optimization software or toolbox. Furthermore, the observer is formulated in a way that is especially efficient for systems with sampled measurement.**

***Index Terms*—Nonlinear observer, numerical method.**

I. INTRODUCTION

Observer design is an essential component in many control applications such as output regulation [6], and cryptography and synchronization [3]. An observer design method is based on an observer gain derived offline such as the observer normal form [4], [14], [15], [17], the high gain observer [9], and the backstepping observer [16]. Another method is based on an on-line optimization approach such as moving horizon observers [1], [2], [13], [18], [22], and the observer using Newton's method [19], [24]. For the former approach, the observer has the advantage of simple online computation if an observer gain can be derived offline. However, the derivation of an observer gain could be cumbersome and difficult, if possible, for highly nonlinear systems. The online optimization method has several different approaches, such as [19], [18], and [24]. In this case, the state is estimated by minimizing the norm of the difference between the observer output and the measured output. Its advantage is the capability of dealing with a variety of properties that are undesirable and troublesome for many existing methods of observer design, such as systems not uniformly observable, time-varying systems, chaotic systems, and systems with unknown parameters. In addition, the observer is portable. Changing a system model does not require a redesign of the observer. In recent work [11] it is shown that a moving horizon observer performs better than EKF for some nonlinear systems with time-varying unknown parameters. However, a main drawback of this approach is that the observer can be computationally demanding. An idea of reducing computational load, implemented in different ways such as [19], [18], and [24], is to perform a limited number of iterations of optimization in each step. In this effort, the stability analysis is based on an accurate integration of the nonlinear dynamics, and/or continuous measurement of the output. Some approaches require the information of Hessian matrix, which is computationally expensive.

In this note, we develop a design framework, including both error estimation and computational implementation, of moving horizon observers. While some similar ideas of moving horizon observers have been studied before ([7], [21]), the results in this note are different. We do not assume accurate solution for either ordinary differential equations (ODE) or nonlinear optimization. In the error estimation, we take into consideration both the integration error and the optimization error. In addition, the observer design is not associate to a fixed numerical method like in [19] and [24]. Instead, the framework developed in this

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