A Pseudospectral Method for the Optimal Control of Constrained Feedback Linearizable Systems

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***Abstract*—We consider the optimal control of feedback linearizable dynamical systems subject to mixed state and control constraints. In general, a linearizing feedback control does not minimize the cost function. Such problems arise frequently in astronautical applications where stringent performance requirements demand optimality over feedback linearizing controls. In this paper, we consider a pseudospectral (PS) method to compute optimal controls. We prove that a sequence of solutions to the PS-discretized constrained problem converges to the optimal solution of the continuous-time optimal control problem under mild and numerically verifiable conditions. The spectral coefficients of the state trajectories provide a practical method to verify the convergence of the computed solution. The proposed ideas are illustrated by several numerical examples.**

***Index Terms*—Constrained optimal control, pseudospectral, nonlinear systems.**

I. Introduction

It is well known [7], [8], [47] that it is extremely difficult to analytically solve a state- and control-constrained nonlinear optimal control problem. The main difficulty arises in seeking a closed-form solution to the Hamilton–Jacobi equations, or in solving the canonical Hamiltonian equations resulting from an application of the Minimum Principle. Over past decades, many computational methods have been developed for solving nonlinear optimal control problems. For instance, in [8], various numerical methods such as neighboring extremal methods, gradient methods and quasilinearization methods are discussed in detail. An update on these methods, along with extensive numerical results is presented in [7]. In [28], a generalized gradient method is proposed for constrained optimal control problems. In [11], the feasibility and convergence of a modified Euler discretization method is proved. A unified approach based on a piecewise constant approximation of the control is proposed in [23].

Numerical methods for solving nonlinear optimal control problems are typically described under two categories: direct methods and indirect methods [3]. Historically, many early numerical methods were based on finding solutions to satisfy a set of necessary optimality conditions resulting from Pontryagin's Maximum Principle [3][35]. These methods are collectively called indirect methods. There are many successful implementations of indirect methods including launch vehicle trajectory design, low-thrust orbit transfer, etc. [3], [6], [9]. Although indirect methods enjoy some nice properties, they also suffer from many drawbacks [2]. For instance, the boundary value problem resulting from the necessary conditions are extremely sensitive to initial guesses [8][2]. In addition, these necessary conditions must be explicitly derived—a labor-intensive process for complicated problems that requires an in-depth knowledge of optimal control theory.

Over the last decade, an alternative approach based on discrete approximations has gained wide popularity [27], [2], [13], [15], [26], [46] as a result of significant progress in large-scale computation and robustness of the approach. The essential idea of this method is to discretize the optimal control problem and solve the resulting large-scale finite-dimensional optimization problem. These types of methods are known as direct methods whose roots can be traced back to the works of Bernoulli and Euler [39]. The simplicity of direct methods belies a wide range of deeply theoretical issues that lie at the intersection of approximation theory, control theory and optimization. Regardless, a wide variety of industrial-strength optimal control problems have been solved by this approach [2], [30], [33], [37].

Considering the widespread use of discrete approximations, it might appear to a novice that theoretical questions regarding the existence of a solution and convergence of the approximations have been answered satisfactorily. While this is somewhat true of Eulerian methods [29], [12], [13], [35], [11] corresponding results for higher-order methods are not only absent, but a number of interesting results are reported in the literature. For example, Hager [26] has shown that a "convergent" Runge–Kutta method does not converge to the continuous optimal solution despite the fact that it satisfies the standard conditions in the Butcher tableau. On the other hand, Betts *et al.* [1] show that a nonconvergent Runge–Kutta method converges for optimal control problems. Thus, it is not surprising that even for Eulerian methods, significant restrictions and assumptions are necessary for proofs of convergence, particularly for state-constrained problems [13]. Note that these issues are quite different from those that were raised in the early days of optimal control as summarized in [35]. While Eulerian methods are widely studied and useful for a theoretical understanding of discrete approximations, they are not practical for solving industrial strength problems, especially the so-called multiagent problems that require real-time solutions

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