H_∞ Control via Measurement Feedback for General Nonlinear Systems

Alberto Isidori, *Fellow, IEEE,* and Wei Kang, *Member, IEEE*

***Abstract*—This paper shows how the problem of (local) disturbance attenuation via measurement feedback, with internal stability, can be solved for a nonlinear system of rather general structure. The solution of the problem is shown to be related to the existence of solutions of a pair of Hamilton–Jacobi inequalities in n independent variables, which are associated with state-feedback and, respectively, output-injection design. The results of the paper extend a number of recent achievements in this area.**

I. Introduction

The development of a systematic analysis of the nonlinear equivalent of the H_∞ (sub)optimal control problem was initiated by the important contributions of Ball–Helton [2], Basar–Bernhard [4], and Van der Schaft [10]. In particular, [10] has shown that the solution of the problem in question in the case of full information configuration, that is when the set of measured variables which are available for feedback includes the state of the controlled plant and the exogenous disturbance input, can be determined from the solution of a Hamilton–Jacobi equation, which is the nonlinear version of the Riccati equation considered in analysis of the H_∞ (sub)optimal control problem for linear systems. More recent contributions to this area of research are the works [11], [6], [3], and [7]. In particular, [6] presents a set of sufficient conditions for the solution of the problem of (local) disturbance attenuation in the case of measurement feedback, that is when the set of measured variables is just a function of the state of the plant and of the disturbance input. The paper [3], among many other important contributions, discusses a number of issues related to the necessity of the (sufficient) conditions proposed in [10] and [6]. The paper [7] considers, as the paper [6] does, systems modeled by equations which are affine in both control and disturbance inputs, and shows that, if a local solution is sought, the sufficient conditions given in [6] can be simplified and turned into conditions involving two Hamilton–Jacobi inequalities in n independent variables, associated with the design of a "state-feedback" and, respectively, of an "output-injection gain."

Manuscript received February 12, 1993; revised September 24, 1993 and April 13, 1994. Paper recommended by Associate Editor, A. Saberi. This work was supported in part by MURST, by the National Science Foundation under Grant ECS-9208306, and by the Air Force Office of Scientific Research under Grant 91-0266.

A. Isidori is with the Department of Systems Sciences and Mathematics, Washington University, St. Louis, MO 63130 and the Dipartimento di Informatica e Sistemistica, Università di Roma "La Sapienza," 00184 Rome, Italy.

W. Kang is with the Department of Systems Sciences and Mathematics, Washington University, St. Louis, MO 63130 USA.

IEEE Log Number 9408275.

In the present paper we study the H_∞ (sub)optimal control problem for systems modeled by equations which are not necessarily affine in the inputs, a more general class nonlinear systems already considered in [3] (which discusses necessary conditions and a separation principle in the case of measurement feedback) and in [12] (which discusses the case of state feedback). More precisely, we present a (more general) necessary condition for the existence of a solution to the problem in the case of measurement feedback and we show that, if this condition is strengthened in a suitable way, then the construction of a feedback law yielding local disturbance attenuation with internal stability becomes possible. The analysis of the necessity extends the results of [3] while the analysis of the sufficiency extends the results of [7].

Consider a nonlinear system modeled by equations of the form

$$\begin{aligned}\dot{x} &= F(x, w, u)\\ z &= Z(x, u)\\ y &= Y(x, w).\end{aligned} \tag{1}$$

The first equation of this system describes a plant with state x, defined on a neighborhood X of the origin in $\mathbb{R}^n$ with control input $u \in \mathbb{R}^m$ and subject to a set of exogenous input variables $w \in \mathbb{R}^r$ which includes disturbances (to be rejected) and/or references (to be tracked). The second equation defines a penalty variable $z \in \mathbb{R}^s$, which may include a tracking error, as well as a cost of the input u needed to achieve the prescribed control goal. The third equation defines a set of measured variables $y \in \mathbb{R}^p$, which are functions of the state plant x and the exogenous input w. The mappings $F(x, w, u)$, $Z(x, u)$, and $Y(x, w)$ are smooth mappings (i.e., mappings of class C^k for some sufficiently large k) defined in a neighborhood of the origin in $\mathbb{R}^n \times \mathbb{R}^r \times \mathbb{R}^m$. We assume also that $F(0, 0, 0) = 0$, $Z(0, 0) = 0$ and $Y(0, 0) = 0$.

The control action to (1) is to be provided by a controller, which processes the measured variable y and generates the appropriate control input u, and is modeled by equations of the form

$$\begin{aligned}\dot{\xi} &= \eta(\xi, y)\\ u &= \theta(\xi, y)\end{aligned} \tag{2}$$

in which ξ is defined on a neighborhood Ξ of the origin in $\mathbb{R}^\nu$ and $\eta : \Xi \times \mathbb{R}^p \to \mathbb{R}^\nu$, $\theta : \Xi \times \mathbb{R}^p \to \mathbb{R}^m$ are smooth functions, satisfying $\eta(0, 0) = 0$ and $\theta(0, 0) = 0$.

The purpose of the control is twofold: to achieve closed-loop stability and to attenuate the influence of the exogenous input w on the penalty variable z. Following numerous authors (see,