reference so as any l_2 sequence can be considered. If, however, a rational model of the reference is provided, the solution can further elaborated and expressed in a polynomial closed-form.

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Nonlinear H_∞ Control and Its Application to Rigid Spacecraft

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***Abstract*— The problem of H_∞ control of a rigid spacecraft with three control torques and disturbances is addressed. The Hamilton–Jacobi–Isaacs inequality for H_∞ feedback design is solved. An H_∞ suboptimal feedback is given, and the stability properties of the closed-loop system are studied.**

I. Introduction

For the last few years, nonlinear H_∞ control method has been extensively developed by many authors (see [8] and [12]). The problem is solved based on solutions of Hamilton–Jacobi–Isaacs inequalities, which are nonlinear partial differential inequalities. In general, solving these inequalities is difficult. The Hamilton–Jacobi equations for certain optimal control problems defined on Lie groups can be solved (see, for instance, [1]). In this paper, the model of a rigid spacecraft, with disturbance inputs, controlled by three control torques is considered. The state space is the tangent bundle of the Lie group $SO(3)$. In Section III, it is proved that the partial differential inequalities associated with H_∞ control method can be solved. Then, an H_∞ suboptimal feedback is designed and its stability properties are studied.

Manuscript received February 11, 1994; revised December 16, 1994.

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IEEE Log Number 9411661.

Stabilizing feedbacks of a spacecraft with or without disturbance inputs have been studied for a long time (see, for instance, [2]–[7], [11], [14], [10], and [13]). In [5], the linear H_∞ method was applied to the linearization of a space-station model. Instead of linearization argument, we apply nonlinear H_∞ control theory to the spacecraft model in this paper. The designed feedback can be interpreted as a combination of linear springs and damping. We can give a geometric description of the limit sets of all trajectories. A subset of the domain of attraction can be found. The angular velocity is globally asymptotically stable under the designed feedback. Therefore, unlike most local stabilization feedback designed by linearization method, the global behavior and the domain of attraction of the closed-loop system under the feedback given in this paper are more visible.

II. The H_∞ Feedback

Let $SO(3)$ be the Lie group of orthogonal matrices with determinant 1, i.e., $R \in SO(3)$ if and only if $R^TR =$ identity and $\det(R) = 1$. Any $R \in SO(3)$ is considered as a rotation matrix which transforms an inertially fixed set of orthonormal axes e_1, e_2, e_3 into a set of orthonormal axes r_1, r_2, r_3 of same orientation and fixed in the spacecraft. The origin of both frames are at the center of mass. The following model of a rigid spacecraft is a vector field on $TSO(3)$, the tangent bundle of $SO(3)$

$$\begin{aligned} \dot{R} &= S(\omega)R \\ J\dot{\omega} &= S(\omega)J\omega + (u_1, u_2, u_3)^T + (w_1, w_2, w_3)^T \end{aligned} \tag{2.1}$$

where J is the inertia matrix of the spacecraft, ω is in $I\!R^3$, $\sum_{i=1}^3 \omega_i r_i$ is the angular velocity, $u = [u_1, u_2, u_3]^T$ is the control input, $w = [w_1, w_2, w_3]^T$ is the disturbance input. The cross product matrix $S(\omega)$ is

$$S(\omega) = \begin{bmatrix} 0 & \omega_3 & -\omega_2 \\ -\omega_3 & 0 & \omega_1 \\ \omega_2 & -\omega_1 & 0 \end{bmatrix}.$$

In the next section, the vector field given by (2.1) is denoted by

$$F(R, \omega) + g_1 w + g_2 u.$$

As we know, the Killing form on the Lie algebra $so(3)$ induces a Riemann metric on the compact Lie group $SO(3)$. Denote it by $d(\cdot, \cdot)$. It is well known that for any matrix $R \in SO(3)$ there exists a vector $\bar{\omega} \in I\!R^3$ such that $R = \exp(S(\bar{\omega}))$. The vector $\bar{\omega}$ is not unique. If it is the one with smallest norm, then the distance between R and the identity matrix I under the induced Riemann metric is

$$d(R, I) = \|\bar{\omega}\|.$$

The H_∞ feedback designed in this section attempts to keep the attitude of the spacecraft aligned with the reference frame, i.e.,

$$\begin{aligned} (r_1, r_2, r_3) &= (e_1, e_2, e_3) \\ \omega &= 0 \end{aligned} \tag{2.2}$$

in the presence of disturbances. For this purpose, we define the penalty function by

$$z(R, \omega, u) = (h(R, \omega), u)^T \tag{2.3}$$