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Calculation of the Minimal Dimension kth-order Robust Servoregulator

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***Abstract*—The design of a minimal dimension kth-order robust servoregulator requires calculation of the minimal polynomial of a class of matrices defined by the given exosystem. A characterization of the minimal polynomial of this class of matrices was given recently in [8] when the given exosystem is semisimple. This paper will further provide a characterization of this class of matrices in the general case. This result leads to a straightforward and efficient procedure to calculate the minimal dimension kth-order robust servoregulator.**

***Index Terms*—Linear algebra, nonlinear control, robust nonlinear servomechanism.**

I. INTRODUCTION

Feedback design for the servomechanism problem can be challenging when a system is highly nonlinear. Methods of finding the kth-order approximate solution of the problem for nonlinear systems with or without uncertainty are introduced in [9] and [6]. The notion of kth-order robust control was proposed in [6] to approximately solve the robust nonlinear servomechanism problem (or robust output regulation). Later it is shown that the kth-order robust control can actually solve the robust nonlinear servomechanism problem under additional assumptions [7], [3].

It was shown that the kth-order robust nonlinear servomechanism problem can be solved by a linear controller that contains an internal model of a dynamic system called k-fold exosystem, a nonlinear analog of the well-known *internal model principle* of linear regulation theory. Therefore, the design of a minimal dimension kth-order robust controller requires the calculation of the minimal polynomial of the k-fold exosystem. The k-fold exosystem is a linear autonomous system generated by the linearized exosystem. A characterization of the minimal polynomial of the k-fold exosystem was given recently in [8] where the given exosystem is semisimple. This paper will further provide a characterization of the k-fold exosystem in the general case. This result leads to a straightforward and efficient procedure to calculate the minimal dimension kth-order robust servoregulator.

The matrix in the k-fold exosystem is also the matrix of a linear differential operator, which arises in some related control problems such as nonlinear optimal control, nonlinear H_∞ control, and feedback linearization. The result in this paper is believed to be helpful to understand the nature of the approximate solutions to these problems.

The rest of this paper is organized as follows. Section II summarizes the results on the nonlinear robust servomechanism theory following the lines of [6]. Section III gives a characterization of the minimal polynomial of the k-fold exosystem. In Section IV, we close this paper with some remarks.

Manuscript received March 6, 1995; revised June 13, 1996. The work of the second author was supported in part by the Hong Kong Research Grants Council under Grant CUHK 380/96E.

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Publisher Item Identifier S 0018-9286(97)02041-2.